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On the Resistance and Electromotive Forces of the Electric Arc

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X. *On the Resistance and Electromotive Forces of the Electric Arc.*

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[Plate 2.]

Since Davy's discovery of the electric arc, a century ago, down to the present time, the nature of the physical processes going on in it, and the mechanism by which it conducts electricity, have been the subject of almost uninterrupted discussion and experiment. In order to explain the fact that the equation connecting P.D. current and length appears to contain a large practically constant term, experimenters have assumed that the arc possesses resistance and E.M.F., though which of the two is the more important in obstructing the flow of the current, or whether both must be considered, has been, and is still, a matter of controversy, the settlement of which, it is hoped, will be furthered by the experimental results described in this communication.

A priori it is highly probable that the resistance and E.M.F.'s of the electric arc, if they exist, will be functions of the current; it is therefore necessary to first consider the definitions of these quantities, as it will largely depend on the definitions adopted whether the arc can be said to possess a resistance, an E.M.F., or neither. The ordinary text-book definitions of resistance and E.M.F. generally start with the assumption that they are constant quantities independent of the current flowing, and their possible variation is generally developed as a secondary effect due to the current altering the state or nature of the body or apparatus considered, these alterations in the state being the primary cause in the change in resistance and E.M.F. observed.

There is much experimental evidence to support the view that when a steady current, A, flows through *any* conducting apparatus, the potential difference, V, between its terminals can be written $V = E + RA$ when E and R, the E.M.F. and the resistance, only depend on the nature, state and movement of the apparatus, and are not directly functions of the current A or the potential V; so that the equation connecting the P.D. and current for any apparatus *under perfectly constant conditions* only contains the current raised to the powers 0 and 1, and the coefficients of these terms are considered as constant specific properties of the conducting apparatus under the given conditions. It is, however, quite conceivable that conductors might exist for which the equation might contain other powers of the current, and in this case their coefficients would be equally justly considered as specific properties of the conductor for which at present no names exist. Is the arc such a conductor ?

On the assumption that *when the conditions are maintained constant* the P.D. can be represented by an equation of the form $V = E + RA$, for all values of the current, then the power spent or furnished by the apparatus $VA = EA + RA^2$, that is, it consists of two parts—the one depending on A^2, and therefore irreversible, so that the sign of the power does not change with change of sign of the current, and the other depending on the first power of the current, and therefore a reversible phenomenon, so that if the apparatus absorbs energy when the current flows in one direction, it will give back energy when the current flows in the opposite direction.

This idea of distinguishing E.M.F.'s from resistances, according to whether the dissipation or absorption of energy is a reversible or irreversible phenomenon, is by no means new, as it underlies the views expressed by Professor FITZGERALD[*] and GRAY[†] in the letters they contributed to the 'Electrician' in the discussion of Messrs. FRITH and RODGERS' paper,[‡] and has also been suggested by Professor S. P. THOMPSON. It seems to afford a satisfactory basis for a definition of resistance and E.M.F., which will be adopted in this communication.

Definition.

Suppose any apparatus under any given set of conditions, through which a certain current is flowing, and that it is required to determine its resistance and E.M.F. under these particular conditions and for this particular current. The energy transferred electrically between the source and the apparatus can be divided into two parts: the one an irreversible part, so that if the direction of the current be conceived reversed the direction of the transfer of energy remains unchanged, and the other a reversible part. If it be found that the irreversible transfer of energy is proportional to the square of the current, and the reversible to the first power of the current, when in some way or other perfect constancy is maintained in all the conditions of the apparatus, such as the size, shape, nature, temperature, temperature gradients, relative movements, &c., of the different parts of the apparatus which are existing with the particular current and under the given set of conditions, then the irreversible rate of transfer of energy divided by the square of the current will be defined as the resistance, and the reversible rate of transfer of energy divided by the first power of

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[*] 'The Electrician,' 1896, vol. 37, pp. 386, 489.

[†] 'The Electrician,' 1896, vol. 37, p. 452.

[‡] 'Proc. Phys. Soc.,' 1896, vol. 14, p. 307.

the current will be defined as the E.M.F. of the apparatus, under the given set of conditions and for that particular current which was initially supposed to be flowing.

In this definition the qualification, "if the irreversible transfer of energy is proportional to the square of the current and the reversible to the first power of the current," predetermines that the apparatus must obey OHM'S law over whatever range it may be possible to vary the current *without in any way changing the state* of the apparatus, in order that it can be said to have a resistance at all. If, however, the qualification is in any case not fulfilled, it will become necessary to consider the terms in the conceivable equation between V and A other than those in which A occurs to the powers 0 or 1.

So far nothing has been said of the signs which the two quantities resistance and E.M.F. can have, as their signs are more or less a matter of convention. If we call a current flowing round the circuit in the same direction as the E.M.F. of the source would tend to make it flow a + current, and a transfer of energy from the source to the apparatus a + transfer of energy, then their signs are determined and agree with ordinary practice, so that the resistance and the E.M.F. of the apparatus which oppose the flow of the current will have + signs. It is to be noticed, however, that this definition does not preclude in any way the possible existence of a negative resistance; for if, instead of an irreversible transfer of energy from the source to the apparatus, proportional to the square of the current, there were found (the conditions of the apparatus being, of course, maintained constant as before) to be a transfer in the opposite direction, *i.e.*, from the apparatus to the source, then the coefficient of A^2 would have to be negative, so that in this case the apparatus would possess a true negative resistance. Although in what follows it will be shown that this is not the case with the arc, it is as well to draw attention to the matter, as a considerable part of the controversy on the negative resistance of the arc under certain conditions arose from some of those who took part defining resistance as an essentially positive quantity, and then trying to prove that it could not be negative in the case of the arc.

A single value of V corresponding to a single value of A is evidently not sufficient to determine whether any conductor fulfils the above definition of resistance and E.M.F. To determine this the current must be varied over some range, δA, and in such a manner that the conditions of the conductor remain unchanged, and a series of observations must be made within this range.

The essential stipulation, that the test must not alter the body tested, is the main difficulty in the experiments on the resistance and E.M.F.'s of the arc. For it is well known that, corresponding with each steady value of the current, the size and configuration of the vapour column and craters are different in spite of the fact that the length, the nature of the electrodes, and the other conditions may be kept constant, so that the arcs corresponding with any two different steady values of the current, however nearly equal they may be, are really two distinct and different phenomena. Therefore all methods which depend on the steady change δV in the potential difference V produced by a given steady change δA in the current A, that is to say, which depend on an excursion on the steady curve between V and A, however small it may be, simply measure the difference between the P.D.'s required to maintain an arc with a current A and a distinct and different arc with a current $A \pm \delta A$, which is evidently no measure of either the resistance of the arc with current A or with $A \pm \delta A$.

If the measuring current δA is only applied for a short time δt, it is necessary that the energy supplied to or removed by it shall be so small as not to appreciably alter the thermal conditions of the very small mass of gaseous and other material which is taking part in the conduction of the current. How extremely short the time that may elapse is will appear later; for the present it is sufficient to point out that it has been found that even in $\frac{1}{10,000}$ second a change of 3 per cent. in the arc current has appreciably altered the thermal conditions and the light emitted by the arc.*

It thus appears that the only available methods of experimentally determining the resistance and E.M.F. of the arc must be based on making the necessary change in the main current, *i.e.*, the measuring current, as small as possible, and on completing the test so soon after making this change that none of the conditions of the arc will have had time to appreciably alter before it is completed.

The first method tried consisted in sending the oscillatory discharge from a condenser through the arc, and recording by means of an oscillograph the variations in the P.D. between the terminals of the arc and in the current through it. If the frequency of the oscillatory discharge can be made so high that the conditions of the arc are not in any way altered by it, then the wave-forms of the oscillatory part of the P.D. and current will be similar curves and in phase if the arc possesses a true resistance. This was not found to be the case with the oscillations used, which had frequencies up to 5000 ∽ per second, the current oscillation always lagging behind the P.D. oscillation.

At low frequencies and with solid carbon electrodes the oscillations were 180° out of phase, and this difference was gradually reduced with increase in frequency to below 90° at 5000 ∽ per second, and there were indications that this lag would finally disappear if a much higher frequency were used, so that the conditions of the arc were not altered by the oscillations.

A large number of experiments, some of which have been published in a paper† before the Institution of Electrical Engineers, were made to determine the effect of small rapid variations in current on the conditions of the arc itself. The conclusion drawn from the above experiments was that a very much higher frequency than 5000 ∽ per second was necessary in order that the arc might not be affected by the measuring current.

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* 'Journal of the Institution of Electrical Engineers,' 1901, vol. 30, p. 236.

† *Ibid.*

Owing to various reasons, the above method was not suitable for these higher frequencies; consequently a new method was devised similar to that used by Messrs. FRITH and RODGERS,* based on the R.M.S. values of the superimposed P.D. and current, and not on the instantaneous values of these quantities, with this difference from Messrs. FRITH and RODGERS' method, that there was a criterion, when a result was obtained, as to whether the arc was behaving like an ordinary resistance or not.

Basis of Method Adopted.

Consider any apparatus A, fig. 1, which may have resistance and E.M.F. but no self-induction or capacity, through which a steady direct current may be flowing, and

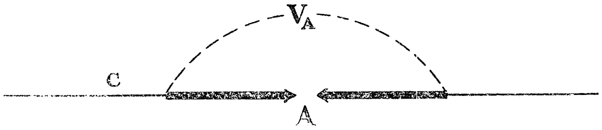

Fig. 1.

let there be mixed with the direct current an alternating testing current of R.M.S. value C.

Let V_A be the R.M.S. value of the alternating part of the P.D. between the terminals of A, and let v_A and c be the instantaneous values of these latter quantities.

The impedance of the apparatus A $= \sqrt{\left(\frac{1}{T}\int_0^T v_A^2\,dt\right)} \Big/ \sqrt{\left(\frac{1}{T}\int_0^T c^2\,dt\right)} = V_A/C = I_A$.

Suppose that the frequency of the alternating current can be made such that the conditions of the apparatus are not in any way changed by the alternating current, then if the apparatus has a true resistance it will be a constant, so that the instantaneous values v_A and c will have a constant ratio, *i.e.*, will obey OHM's law. Then the wave-forms of V_A and C will be similar curves and in phase, and the true resistance of the apparatus $= v_A/c = V_A/C = I_A$.

A criterion is now required that v_A and c do obey OHM's law, and this is supplied by the power-factor of the apparatus A, the power-factor being defined as

$$\frac{1}{T}\int_0^T v_A c\,dt \Big/ \sqrt{\left(\frac{1}{T}\int_0^T v_A^2\,dt\right)}\sqrt{\left(\frac{1}{T}\int_0^T c^2\,dt\right)}.$$

For it can be proved that the necessary and sufficient condition that the power-factor may be unity is that the wave-forms of V_A and C are similar curves and in phase, so that v_A and c obey OHM's law.

Therefore if, when the current C and its frequency are such that the conditions of the apparatus are not changed, it can be proved that the power-factor of A is unity, then A has a true resistance numerically equal to I_A. Further, in any apparatus in

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* 'Proc. Phys. Soc.,' 1897, vol. 14, p. 307.

which the resistance is a function of the conditions, the possibility of obtaining the power-factor unity is a proof of the constancy of the resistance and consequently of the conditions, so that if the apparatus A is an arc, and if it can be shown that a sufficiently high value of the frequency can be reached for which the power-factor is unity, then the conditions of the arc are not being altered by the alternating testing current, and the arc has a true resistance numerically equal to I_A.

It is assumed above that the arc or apparatus A has no self-induction or capacity; to prove this it must be shown that not only can the frequency be increased till the power-factor becomes unity, but also that it remains so for a considerable further increase of frequency.*

Finally, therefore, in order to prove that the arc has a true resistance, and to find its value, it is necessary to show:—First, that it is possible to find a value of the frequency of the alternating testing current for which the power-factor of the arc with respect to this current is unity; second, that the power-factor remains unity and the impedance constant even when the frequency is greatly increased above this value; third, to determine the value of the impedance of the arc under these conditions, which will also be its true resistance.

Method of Measuring the Impedance and Power-Factor.

At first sight it would seem as if there were a considerable number of available methods for accurately measuring these quantities. But the number of methods becomes exceedingly limited when it is considered that it is necessary for the alternating testing current C to have as small a R.M.S. value as possible (0·1 ampère was that generally used in the experiments), and that the effects due to this small current have to be sorted out when it is mixed with a direct current of 10 ampères or more. Added to this, to make the difficulties greater, it was finally found necessary to use frequencies up to and even over 100,000 ∽ per second. Wattmeters and dynamometers were tried and abandoned, and finally the well-known 3-voltmeter method† was adopted.

A non-inductive resistance R was placed in series with the apparatus A (fig. 2), through both of which the main direct current flowed; to this direct current there was added, as before, an alternating measuring current of R.M.S. value C.

Let V_A, V_R, and V be the R.M.S. values of the alternating part of the P.D.'s as shown in fig. 2. The impedance of A is $I_A = V_A/C = RV_A/V_R$. Power factor of A is $P_A = (V^2 - V_A{}^2 - V_R{}^2)/(2V_AV_R)$.

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* It seems possible that the power-factor of a conductor which did not possess self-induction or resistance in the ordinary sense of these terms might still depart from unity at very high frequencies, owing to the time taken by the carriers of the electric charge to hand it on becoming comparable with the periodic time of the testing current.

† See AYRTON and SUMPNER, 'Roy. Soc. Proc.,' vol. 49, p. 424.

The two quantities, the impedance and power-factor, are therefore determined in terms of a resistance R and three R.M.S. voltages quite independent of any

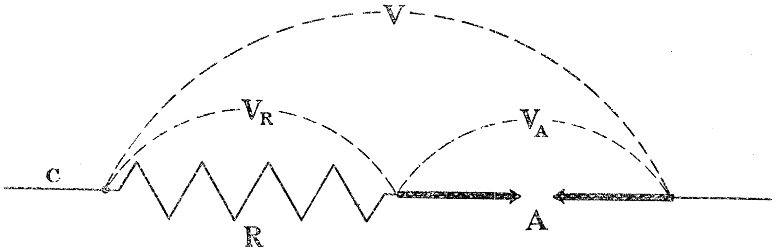

Fig. 2.

assumptions as to the wave-form of the alternating testing current. If the same voltmeter is used to measure each of these voltages, then it will be noticed that the results only depend on the relative calibration of one instrument, a consideration of great importance owing to the difficulties in the way of accurate absolute calibration with the very high frequencies used.

Circuit and Apparatus Used.

In order to measure the impedance and power-factor by the method just considered, several different arrangements of the circuit were tried; that finally adopted for the

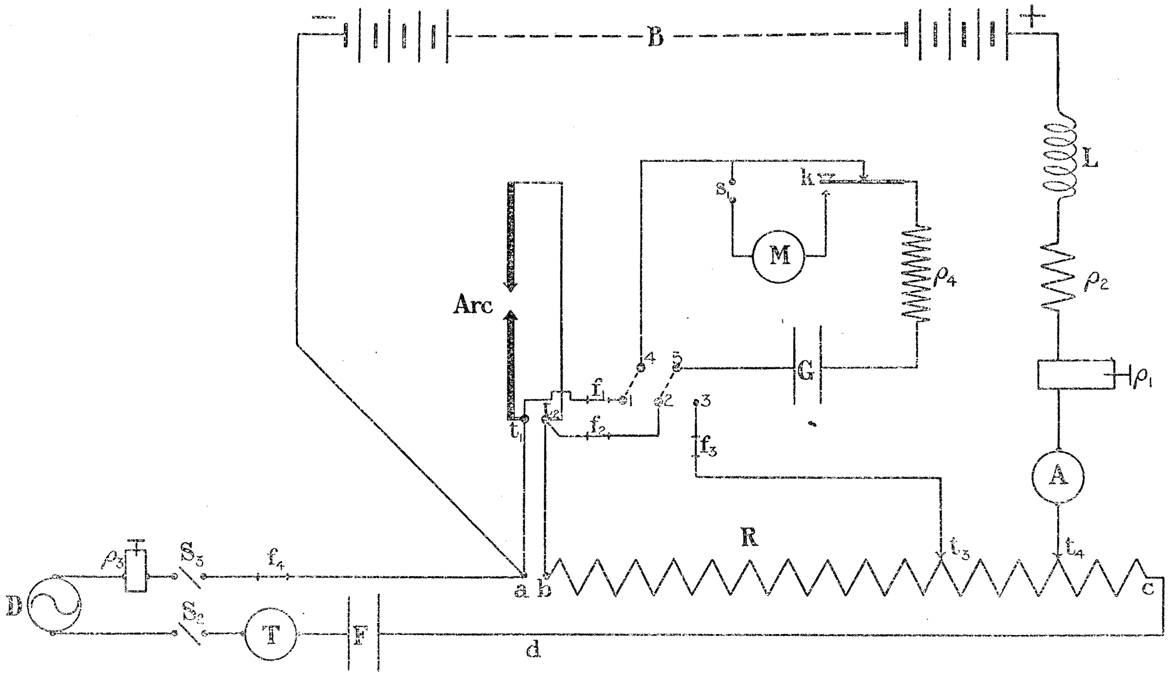

Fig. 3

experiments is shown diagrammatically in fig. 3. The main direct-current circuit consisted of:—

B, a battery of from 50 to 90 accumulators which supplied the direct current to the arc,
L, a self-induction,
ρ_1, ρ_2, adjustable resistances,
A, a Weston ammeter which indicated the direct current through the arc,
R, the standard non-inductive resistance, consisting of 12 coils of about 0·5 ohm each, described later,
t_1 and t_2, the terminals of the arc lamp,
t_3 and t_4, terminals which could be moved along R,
1, 2, and 3, mercury cups in a wax block,
f_1, f_2, and f_3, fine fuses in the connections between the above points.

The direct P.D. between the terminals of the arc lamp (called "P.D. arc lamp") was measured with a Weston voltmeter (not shown) which was connected between the points 1 and 2. The same voltmeter, which could be connected between points 2 and 3, gave direct P.D. between the terminals t_2 and t_3 (called "P.D., R"), from which the resistance of R was obtained in terms of the readings of the voltmeter and ammeter, both of which were carefully standardised.

An image of the arc was projected by means of a lens on to a screen, divided and marked so as to read the actual distance in millimetres between the *ends* of the carbons; this distance is called the "arc length."

The circuit for adding the alternating testing current to the direct current consisted of:—

D, a special high-frequency alternator which supplied the testing current, capable of producing small alternating currents having frequencies up to 120,000 ∽ per second,
ρ_3, a variable resistance,
S_2 and S_3, plug switches,
f_4, a fine fuse,
T, an Ayrton and Perry reflecting twisted-strip ammeter, having a sensibility of 400 scale divisions for 0·1 ampère at a scale distance of 1700 divisions (1 division = $\frac{1}{40}$ inch),
F, a condenser to prevent any direct current from flowing through the alternator, the capacity of which was 1 mf. for the frequencies from 120,000 to 50,000 ∽ per second; 2·5 mf. for frequencies from 50,000 to 2000 ∽ per second; and increased up to 15 mf. at 250 ∽ per second.

The alternating current supplied to the arc circuit was kept at a constant R.M.S. value as read on T by means of the adjustable resistance ρ_3; this current flowed through the arc and R in series, and was practically prevented from flowing through the battery by the self-induction L. At the higher frequencies of from 10,000 to 120,000 ∽ per second this self-induction behaved almost like an insulator; at the lower frequencies a small percentage of the alternating current flowed round the battery side of the circuit; but this did not affect the accuracy of the results, as the expressions for the impedance and power-factor are independent of the current, so that it is only necessary to maintain the testing current constant during each test.

The circuit for measuring the alternating part of the P.D. arc lamp V_A, the P.D. between the terminals of R, V_R, and the P.D. total, V, consisted of:—

M, a thermo-galvanometer, whose deflections were practically proportional to the mean squared value of the current through it, and which, though practically non-inductive, gave a deflection of about 500 scale divisions for 1 milliampère of *alternating* current. The deflections of this instrument were read on the same scale as those of T, so that both could be observed at one time;

ρ_4, an ordinary resistance box,

k, a key,

S_1, a switch, consisting of mercury cups in a wax block,

G, a condenser which allowed a current due to the alternating part of the P.D. to flow through M, but prevented any current due to the direct P.D. from flowing through it,

4 and 5, mercury cups by means of which the measuring circuit could be connected to either the points 1 and 2; 2 and 3; or 1 and 3; to measure V_A, V_R, and V respectively.

The impedance of the measuring circuit, which consists of the thermo-galvanometer M, the resistance ρ_4, and the condenser G, need not be accurately known, as it is only the relative values of V_A, V_R, and V which are required to a high degree of accuracy and not their absolute values. As the frequency in each experiment was kept constant to well within 1 per cent., the impedance of the measuring circuit need not be quite independent of the frequency.

In all arcs neither the direct P.D. nor the current keep quite steady, owing to the necessity of feeding the carbons together, and to the impurities, cracks, &c., existing in them, so that the comparatively slow variations produced must be prevented from sending any appreciable currents through M; for this reason the capacity of G was made as small as compatible with the impedance of the measuring circuit, not depending too much on the frequency. A standard $\frac{1}{3}$ mf. condenser was used for G for all frequencies from 120,000 to 10,000 $\sim$ per second inclusive; down to 3,000 1 mf. was employed, and for all lower frequencies 5 mf. Even under these conditions it was absolutely essential that the battery B should not be in use for any other experiments, or spurious currents were obtained through M, and such a thing as the arc giving a small hiss sent the spot off the scale.

The key *k* was so arranged that in its up position the condenser G was always kept charged to the correct P.D., so that on depressing it G was neither suddenly charged nor discharged through M, for owing to the delicate nature of the latter any considerable sudden change in the P.D. between the armatures of G sent sufficient current through M to burn it up. Even this precaution did not prevent M from being burnt up several times, owing to the battery connections being broken or the arc going out while the key k was depressed.

The switch S_1 was to enable M to be completely disconnected when taking its zero, as it was thought that at these high frequencies, so long as one pole was connected, there might be a small current flowing into the instrument, due to the capacity of the instrument with surrounding objects, but this effect was not observed.

Owing to the very high frequencies used, very great care had to be taken in arranging the circuit so as to avoid self-induction and capacity errors in the leads and connections. All the leads through which the alternating current flows were carefully twisted and bound together.

To reduce any possible error caused by the lead t_2b between the arc and R, the drop along which had to be included either with the arc or with R, this lead was made by twisting together 12 No. 23 double cotton-covered wires, to avoid possible skin effects, and its length was reduced to 60 centims. As first constructed, each wire of the lead t_2b was twisted with the corresponding wire of lead t_1a, which was of the same length and made in the same way, and then the 12 pairs of wires were twisted together. It was found on testing these leads that at a frequency of 18,000 ∽ per second, and with a P.D. of 33 volts between the two leads, an alternating current of about $1 \cdot 9 \times 10^{-3}$ ampère flowed between them due to their capacity. To reduce this capacity current, two new leads were constructed, exactly the same as before, only that instead of twisting the individual wires belonging to each lead together in pairs, all the wires belonging to each lead were stranded together so as to form two separate leads. The lead t_2b was then bound over with a layer of silk tape and the two leads were twisted together. On re-testing in the same way as before, the capacity current was found to be reduced to about $0 \cdot 53 \times 10^{-3}$ ampère with 33 volts between the leads. As the alternating P.D. between the leads was under 0·5 volt in most of the experiments, this capacity current was negligible compared with the working current of 0·1 ampère.

As the self-induction of R had to be determined and allowed for, the lead t_2b was included with R, so that its small self-induction could be corrected for at the same time.

The lead c, d was brought back along the connections b, c between the 12 coils of R, so as to neutralise as well as possible the magnetic field of these connections.

In arranging the circuit the capacity of those parts of the main circuit between the measuring points t_1, t_2, t_3, as well as of the whole of the measuring and alternator circuits, to surrounding objects and to earth, was kept as small as possible, so as to avoid what might be called capacity leaks. The alternator itself was practically insulated from earth by being fixed down to a wooden frame, and the field circuit of the alternator was well insulated and removed from earth.

To form some idea of the magnitude of these capacity currents or leaks, the four points *a*, *b*, *c* and the arc, were opened in pairs. One of the opened points was re-connected through the thermo-galvanometer used as an ammeter to measure the capacity current supplied to that part of the circuit between the instrument and the other point that was opened. The direct current circuit was disconnected at the points *a* and *d*. The alternating currents observed are given in Table I. for a R.M.S. P.D. of 3·65 volts and a frequency of 100,000 ~ per second.

TABLE I.—Capacity Currents in Leads.

Thermo-galvanometer at—	Points.				Alternating current in 10^{-4} ampère.
	a.	*b*.	*c*.	Arc.	
a	—	closed	closed	open	3·8
a	—	,,	open	closed	1·6
b	closed	—	closed	open	2·9
b	,,	—	open	closed	1·2
c	,,	closed	—	open	3·9
c	,,	open	—	closed	1·2

The maximum value of the capacity leak observed is $3·9 \times 10^{-4}$ ampère at the highest frequency used in any series of experiments, and at a P.D. about seven times as high as that used, so that if the capacity current is proportional to the P.D., it should not exceed $0·6 \times 10^{-4}$ ampère, or about 0·06 per cent. of the working current of 0·1 ampère, and may therefore be neglected. Even if this capacity leak had been many times larger, it would not have appreciably affected the P.D.'s measured, since at these high frequencies the arc behaves like a non-inductive resistance, and therefore the measuring current and the capacity current would add approximately as vector quantities at right angles.

As it is only the relative values of V_A, V_R, and V that are required very accurately, any small self-induction in the leads connected with the points 1, 2, and 3, or in the measuring circuit itself, is of no importance. Nevertheless, all the leads were carefully stranded together to prevent any E.M.F.'s being induced in this circuit, caused by magnetic induction. The only wire which could not be stranded with a corresponding wire was about 30 centims. of the lead between the fuse f_3 and the movable contact t_3. Experiments were made by varying its length and position to see if it introduced any error, but none could be detected.

The condensers F and G were placed some distance apart, so as to prevent any direct electrostatic action between them. This, as well as any mutual induction, both electrostatic or magnetic, between any part of the main or alternator circuits and the measuring circuit was examined for, but none could be detected. Experiments were also made to see if the condenser G really prevented M from being deflected by steady direct P.D.'s; for this purpose steady P.D.'s of from 100 to 180 volts were applied to the terminals of the arc, the carbons being separated, and M was connected in the ordinary way as if to measure V_A. A deflection of M might have occurred due to leakage or electrostatic forces, but no such deflection was observed.

The Arc Lamp.

A hand-fed arc lamp, enclosed in an iron case, was used in all the experiments. The sliding contacts were shunted by flexible leads, so as to avoid any uncertainty in the resistance of these contacts. The resistance of the contacts and of the carbons was determined by adjusting the carbon holders to 10 centims. apart as in use, short-circuiting them with a brass rod and the different carbons in turn, and then measuring the drop in volts between the terminals of the lamp when 10 ampères direct current flowed round the frame and holders, etc. The results are given below:—

TABLE II.

Carbon holders short-circuited with	Resistance.
	ohms.
Brass rod	0·0017
11 millims. solid "Conradty Noris" carbon	0·13
11 ,, cored ,, ,, ,, ...	0·22
11 ,, solid "Le Carbon" electrographitic carbon . .	0·048

The self-induction of the loop formed by the frame of the lamp with the carbon holders 10 centims. apart and short-circuited by the brass rod was approximately determined by passing an alternating current round the lamp and measuring the P.D. between its terminals, at a frequency of 30,000 $\sim$ per second. The value obtained was $2{\cdot}4 \times 10^{-7}$ henry, which was used as a correction. All the above tests were taken with the lamp in place in its case, and with the carbon holders, etc., in exactly the positions they occupied when commencing a test on the impedance of the arc.

The Standard Non-Inductive Resistance, R.

The essence of the test of the arc consisted in comparing its behaviour to alternating currents of various frequencies with that of the standard resistance R in series with it. It was, therefore, necessary that R should be as free as possible from self-induction and capacity. The type of resistance adopted was that described by Professor AYRTON and Mr. MATHER before the Physical Society[*] in 1891.

As both the self-induction and the capacity depend on the size of the resistance, it

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[*] 'Philosophical Magazine,' 1892, vol. 33, p. 187.

was decided to make the resistance as small as possible and to allow the temperature of the strips to rise considerably, which necessitated slightly modifying their design. The resistance consists of 12 platinoid strips, each about 170 centims. long, 2·5 centims. wide, and 0·076 millim. thick. Each strip is folded back on itself and has its ends soldered to two brass blocks let into the top of the frame, and is stretched tight with a tension of about 4 lbs. by means of a brass spring attached to a small glass tube, about 5 millims. diameter, at the bottom of the loop formed by the strip. Between the up and down sides of the strips is placed a sheet of asbestos millboard, about $\frac{1}{32}$ inch thick, and the strips are pressed together against this by glass rods from side to side of the frame.

The resistance of each of the 12 strips was roughly adjusted to 0·5 ohm, and the strips could be used in series or parallel by connecting up the brass blocks forming the ends of the strips with copper links and set-screws as required. Owing to the considerable heating of the strips by the current, their resistance depended on the current; thus the resistance of all the strips in series, which was 6·00 ohms with 1 ampère flowing, rose to 6·25 ohms with the current of 10 ampères which was used in many of the experiments. For this reason, and because it formed a check on the instruments, the resistance of R was determined during each experiment from the known values of the direct current and P.D. between its terminals, and the value so obtained was used in calculating the results.

The apparent self-induction of R, including the connection between it and the arc lamp already mentioned, was measured by comparing it with a non-inductive resistance put in place of the arc. This latter resistance (see fig. 4) was made to imitate an arc possessing non-inductive resistance localised between the ends of the carbons. It consisted of 158 millims. of No. 38 platinoid wire bent back on itself, the two extremities being soldered to the ends, previously copper plated, of two solid "Conradty Noris" carbons; these carbons were held in the carbon holders of the lamp so that the resistance wire occupied the position the arc would when burning. A piece of mica was interposed between the ends of the carbons which served to keep the loop of wire taut. This method of determining the correction to be applied to R really converts the test of the arc into a substitution test, for having determined how an ordinary metal resistance behaves when localised between the carbon tips, the behaviour of an arc substituted for it under exactly similar conditions was compared with it.

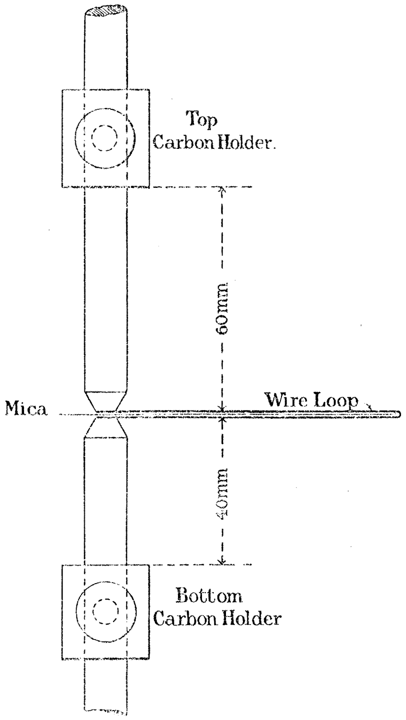

Fig. 4.

To compare R with the wire, they were adjusted to have practically the same resistance. A current, either direct or alternating, having a R.M.S. value of 0·1 ampère as indicated by T, was sent through R and the wire in the place of the arc in series, fig. 3. The P.D. between the terminals of R and between the terminals of the arc lamp (*i.e.*, wire) was measured by means of M, in the same way as in the experiments on the arc, the condenser G being short-circuited. The amount by which the impedance of R exceeded the impedance of the arc lamp and wire is tabulated below for the strips 1 to 7 having a resistance of 3·50 ohms. Each of the results is the mean of at least 12 comparisons. The self-induction of the loop formed by the frame of the arc lamp has been found to be about $2{\cdot}4 \times 10^{-7}$ henry, and the self-induction of the wire itself was calculated to be about 3×10^{-8} henry; so that the total self-induction of the lamp and wire is $2{\cdot}7 \times 10^{-7}$ henry. Assuming that the alternator gives a sine-wave form, which was approximately the case, and allowing for this self-induction and for the difference in resistance 0·03 per cent. between the lamp and R, the true power-factor, $\cos\eta$ of R, has been calculated, from which η the lag of the current in R behind the P.D. and the time constant have been deduced.

TABLE III.—Test of R, Resistance 3·50 ohms.

Frequency ∽ per second.	Impedance of R > impedance of lamp and wire.	$\cos\eta$.	η.	Time constant in 10^{-7} second.	Impedance of R > its resistance.
	per cent.		° ′		per cent.
0 (direct current)	0·03	1·0000	0	—	0
32,400	0·17	$0{\cdot}998_5$	3 $_8$	$2{\cdot}_7$	$0{\cdot}1_5$
41,000	0·25	$0{\cdot}997_6$	5 5_8	$2{\cdot}_7$	$0{\cdot}2_4$
50,700	0·39	$0{\cdot}996_1$	5 $_4$	$2{\cdot}_8$	$0{\cdot}3_9$
60,000	0·51	$0{\cdot}994_8$	5 5_1	$2{\cdot}_7$	$0{\cdot}5_2$
80,000	0·90	$0{\cdot}990_6$	7 5_2	$2{\cdot}_8$	$0{\cdot}9_4$
100,000	1·28 {mean of 24 tests}	$0{\cdot}986_3$	9 3_0	$2{\cdot}_7$	$1{\cdot}3_7$
120,000	1·85	$0{\cdot}980_1$	11 2_7	$2{\cdot}_7$	$1{\cdot}9_9$

Method of Experiment.

The arc length, the direct current, and the frequency of the alternating testing current having been decided upon for any experiment, the test was carried out in the following manner. The carbons having burnt into shape corresponding with the required current and length, a rough experiment on the impedance of the arc was made, and the value of R adjusted by moving t_3 (fig. 3) to that contact on R which made V_A and V_R most nearly equal, as this gives the greatest accuracy in the power-factor by the 3-voltmeter method.

The main direct current was now interrupted and the arc short-circuited by pressing the carbons together. The testing current was adjusted to its working value, in most cases 0·1 ampère, as read on T, and the resistance ρ_4 in series with M was adjusted until V_R gave a deflection of 100 scale divisions on M. The reason that this adjustment was made without the direct current flowing was that it formed a check on the satisfactory working of the measuring circuit, since any apparent change in the sensibility of M when the direct current was re-established would have indicated an error somewhere.

With solid carbons the positive or upper electrode was adjusted to project 6 centims. from its holder and the lower or negative 4 centims. With cored carbons these lengths were 7 and 4·5 centims. respectively. The object of adjusting these lengths was to make the mean resistance and self-induction of the loop formed by the frame of the arc lamp and carbons as nearly as possible the same in every experiment.

The arc was now re-started, and the length and direct current having been adjusted, the carbons were fed together as they burnt away, so as to keep the direct current constant during the whole of the time (about half an hour) that V_A, V_R, and V were being determined. This kept the P.D. arc constant, as long as the P.D. of the battery remained constant; any slight drop in this P.D. was compensated for by adjusting ρ_1, any considerable drop necessitated recommencing the experiment. The arc length with solid carbons also remained constant; but with cored carbons it constantly varied about a mean value according to the amount of material from the core present in the arc. In all cases, readings were only taken when the length was observed to be correct as well as the direct P.D. and current.

As soon as about 5 millims. had burnt off the end of the positive carbon, the deflections of M corresponding to V_A, V_R, and V were observed in turn, the zero of M being taken after each reading, until in most cases five consecutive sets were obtained which were reasonably consistent with one another, the R.M.S. value of the testing current as read by T and its frequency being kept constant. It was easy to obtain individual deflections corresponding with V_R which differed from the mean by less than 0·3 per cent. The deflections corresponding with V_A and V were not so definite, V_A being within 1 per cent. and V within 0·6 per cent. of the mean, except in a few exceptionally unsteady arcs, such as long-cored arcs and small-current arcs.

The values of the direct P.D. arc lamp and direct P.D. R were noted, and the drop of volts in the frame of the lamp and carbons was found by pressing the carbons together, the direct current being so adjusted that when the carbons were in good contact its value was that used for the experiment. This observation was repeated until consistent results were obtained with the carbons hot as in use. By deducting this value from P.D. arc lamp, P.D. arc was obtained.

The relative calibration of the thermo-galvanometer M was then determined by means of direct currents. This completed the observations required for a single experiment.

Mean deflections corresponding with V_A, V_R, and V, were calculated and corrected for the relative calibration of the thermo-galvanometer. From these values the impedance of the arc lamp $I_A = I_R$, V_A/V_R, and the power-factor of the arc lamp, $P_A = (V^2 - V_A^2 - V_R^2)/2V_AV_R$, were calculated.

In order to obtain from these values the impedance and power-factor of the *arc itself*, a small correction had to be applied to I_A for the resistance and self-induction of the loop formed by the frame of the lamp and the carbons, and also to P_A for the self-induction of R which had previously been determined. To make these small corrections, it was necessary to assume that the alternating current had a sine-wave form, which was approximately the case.

As a check on the method of experiment and on the calculation and correction of results, the impedance and power-factor of the platinoid resistance, described on p. 317, which had a resistance of 3·499 ohms and a self-induction of about 3×10^{-8} henry, were determined, the experiment and calculations being performed in the same manner as for the arc. The values obtained were: impedance 3·50 ohms, power-factor 0·999, which show that the method was satisfactory in this case.

Results Obtained by Varying the Frequency.

The fundamental experiment of this investigation into the resistance of the electric arc consists, as has already been explained, in varying the frequency of the superimposed alternating testing current, in order to determine whether with a sufficiently high frequency the condition of the arc will remain unchanged, the value of the resistance being then measured at this frequency. The criterion that the conditions of the arc remain unchanged has been shown to be that the power-factor of the arc as measured with the superimposed alternating current must be unity. The true resistance will then be equal to the impedance.

The results of the experiments on the effect of varying the frequency on the power-factor and the impedance for solid and cored* arcs are represented graphically in Curves I. and II. (Plate 2).

With solid carbons the power-factor at 250 ∽ per second is − 0·91. On increasing the frequency it decreases numerically until it vanishes and changes sign at 1,950 ∽ per second, the waves of superposed alternating P.D. and current being then 90° out of phase. With further increase of frequency the power-factor increases rapidly at first, then more and more slowly, becoming asymptotic to + 1, and finally practically attains this value at a frequency of 90,000 ∽ per second; above this frequency the power-factor is, within the limits of experimental error, equal to + 1 up to the highest frequency attained, namely, 120,000 ∽ per second. The impedance of the solid arc increases with increase of frequency from 0·97 ohm at 250 ∽ to 3·8 ohm at a frequency of 90,000 ∽ per second, above which it remains practically constant.

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* "Solid" and "cored" arc mean respectively arc between two solid carbons and between two cored carbons.

At frequencies above 90,000 the power-factor is + 1, therefore the excursions of the P.D. and current obey Ohm's law, and the impedance of the arc is equal to its true resistance. So that *the true resistance of an arc,* 3 *millims. long, between* 11 *millims. solid "Conradty Noris" carbons, and through which a current of* 9·91 *ampères is flowing, is* 3·81 *ohms.*

The P.D. between the terminals of the arc, accounted for by ohmic drop in the arc, is therefore 37·8 volts out of an observed P.D. arc of 49·8 volts, so that *there appears to be a real back electromotive force opposing the flow of the currents in this arc of* 12 *volts.*

Considering next Curve II. for both *cored* carbons, the power-factor at the lowest frequency of 250 ∽ per second has a positive value of + 0·67 and increases asymptotically, as in the case of solid carbons, until it is practically + 1 at a frequency of 15,000, and remains unity within the limits of experimental errors up to the highest frequency tried of 50,000 ∽ per second, the impedance becoming practically constant, as with solid carbons.

Therefore *the true resistance of an arc* 3 *millims. long, between* 11 *millims. cored "Conradty Noris" carbons, and through which a current of* 10 *ampères is flowing, is* 2·54 *ohms, and the back E.M.F. is* 16·9 *volts,* calculated in the same way as for solid carbons.

Finally, therefore, arcs between either solid or cored carbons have both back E.M.F. and resistance, and the true values of these quantities differ greatly from those usually assigned to them.

In order to test whether the R.M.S. value of the added alternating current affected the values obtained for the impedance and power-factor, the testing current was varied over the range 0·036 ampère to 0·130 ampère, and the impedance and power-factor were found to be constant within the limits of experimental error.

It is of interest to enquire how the results obtained by Messrs. Frith and Rodgers can be explained by the aid of these curves. The quantity that they measured and called the resistance of the arc was really the impedance of the arc and a certain resistance together, less the impedance of the resistance part alone. From fig. 2, p. 311, using the same notation as before and assuming R non-inductive,

$$\mathrm{PVC} = \mathrm{V_R C} + \mathrm{P_A V_A C}, \quad \text{or} \quad \mathrm{PV/C} = \mathrm{R} + \mathrm{P_A I_A}.$$

The quantity measured by Messrs. Frith and Rodgers was V/C − R.

In the case of *solid* carbons at their frequencies, about 100 ∽ per second, $\mathrm{P_A}$ is roughly − 1, and a little consideration shows that under these conditions P was practically + 1, so that the quantity they called the resistance of the solid arc was $\mathrm{P_A I_A}$, or the product of the power-factor into the impedance of the arc, which is evidently a negative quantity for frequencies under 1950 ∽ per second with the solid arc investigated in Curves I. As they did not use the same make or size of carbons as those used in this paper, it is impossible to make an accurate comparison between the results obtained. Taking arcs of the same length and current between "Apostle," "Brush" and "Carré" carbons, the mean of the value of what Messrs. FRITH and RODGERS called the resistance of the arc is about 0·79 ohm, and this agrees very well with the value of $P_A I_A$ obtained by extra-polating Curves I. back to about 100 ∽ per second. So that this curve explains both the sign and the value of the so-called negative resistance of the solid arc.

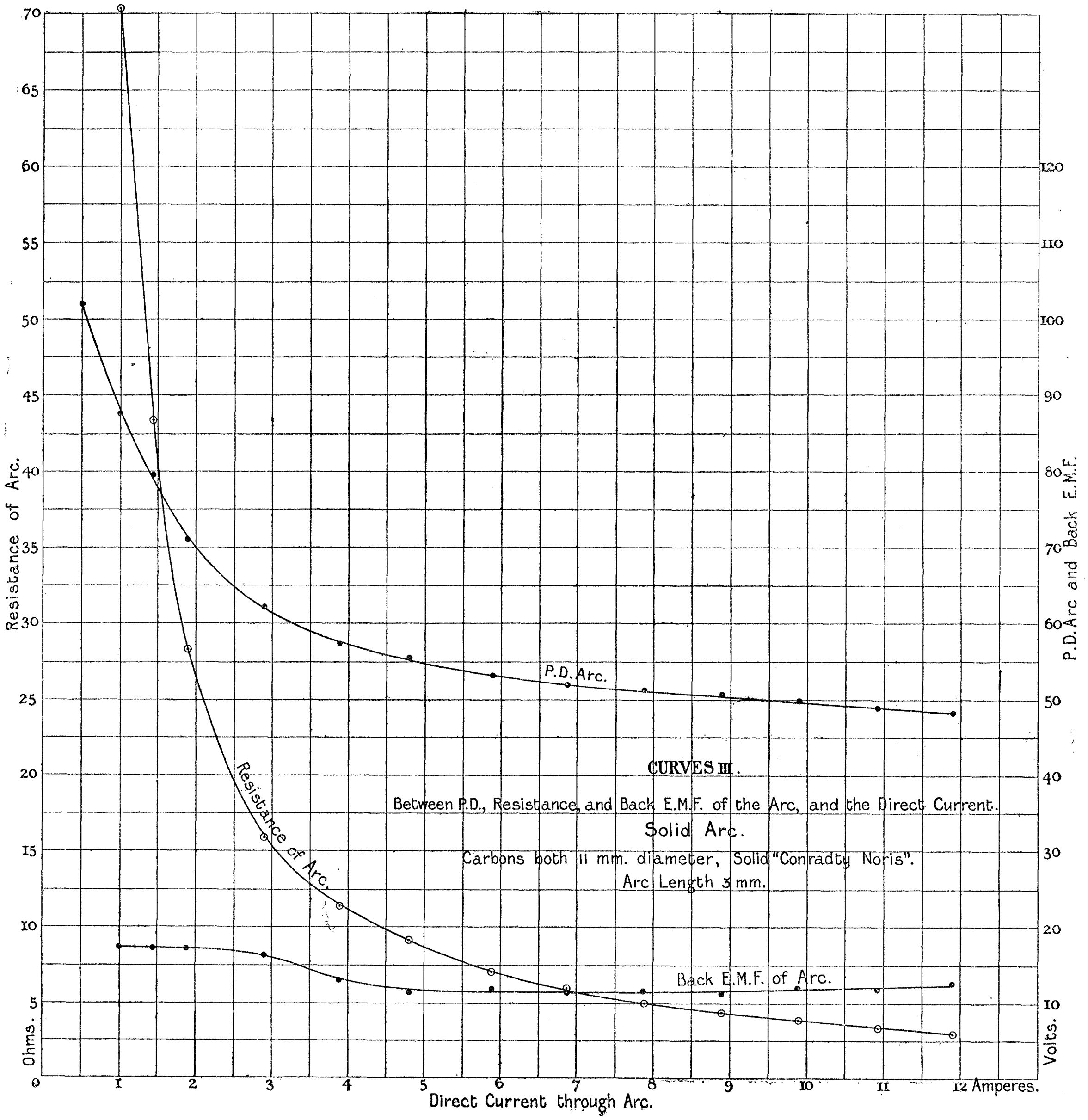
With *cored* carbons, P_A will only be about + 0·5 at 100 frequency. The value of P is unknown, but is certainly less than unity, so that the quantity measured by Messrs. FRITH and RODGERS for cored carbons must be less than $P_A I_A$. By extrapolation from Curves X., the value of $P_A I_A$ at 100 frequency for cored "Conradty Noris" carbons is about + 0·5 ohm. Owing to the indefinite nature of cored carbons, this value cannot be expected to agree very well with those obtained by Messrs. FRITH and RODGERS for carbons of other makers. Taking arcs of the same length, and current as that used in Curves II., the values given by Messrs. FRITH and RODGERS for different makes are:—"Apostle," 0·03 ohm; "Brush," 0·59 ohm; "Carré," 0·55 ohm. The two latter values agree with the value 0·5 of $P_A I_A$ deduced from the curves for "Conradty Noris" carbons to a higher degree of accuracy than might reasonably be expected, whereas the disagreement in the case of "Apostle" carbons, as measured by Messrs. FRITH and RODGERS, seems to indicate that their result is in some way abnormal, possibly due to some accidental impurity in the core.

The critical frequency of 1·8 ∽ per second observed by MESSRS. FRITH and RODGERS for the cored arcs, at which the quantity they measured changes sign, evidently corresponds with the power-factor changing sign, so that the curve between power factor and frequency would cut the zero line at a frequency about 1·8 per second, and it is evident that at lower frequencies it would become negative, an increase of current being accompanied by a decrease in P.D. The curves for the cored arc, if they could be obtained down to sufficiently low frequencies, would therefore be similar to those obtained for the solid arc. The great difference between the solid and cored arcs lies in the much greater quickness with which the conditions of the former can vary, corresponding to any change, however small, in the current through the arc. The relative sluggishness of the cored arc is probably due to the presence in its vapour column of saline matter derived from the core.

The fact that the solid arc has a negative power-factor at frequencies below the critical frequency of 1950 ∽ indicates that the arc is under these conditions supplying power to the alternating current circuit, and that this is the fact can easily be shown experimentally by connecting a wattmeter so as to measure the power supplied to the solid arc by the alternating current, when it will be found that at low frequencies *the solid arc is actually supplying power to the alternate-current circuit, while at frequencies above the critical value the alternate-current circuit supplies power to the arc.* This observation is of course not in any way at variance with the principle of conservation of energy, since the alternating energy given out by the arc is derived from the direct-current energy supplied to it, the arc acting as a converter. This fact, that the solid arc is capable, under suitable conditions, of automatically transforming energy derived from a source of direct current into alternate-current energy of any frequency over a wide range, is the explanation of the transformation of energy observed in the Musical Arc recently shown for the first time at the Institution of Electrical Engineers.*

Effect of Varying the Direct Current.

Having found that with a sufficiently high frequency it was possible to measure the true resistance of the arc, this resistance was measured for arcs of a fixed length



CURVES III.
Between P.D., Resistance, and Back E.M.F. of the Arc, and the Direct Current.
Solid Arc.
Carbons both 11 mm. diameter, Solid "Conradty Noris".
Arc Length 3 mm.

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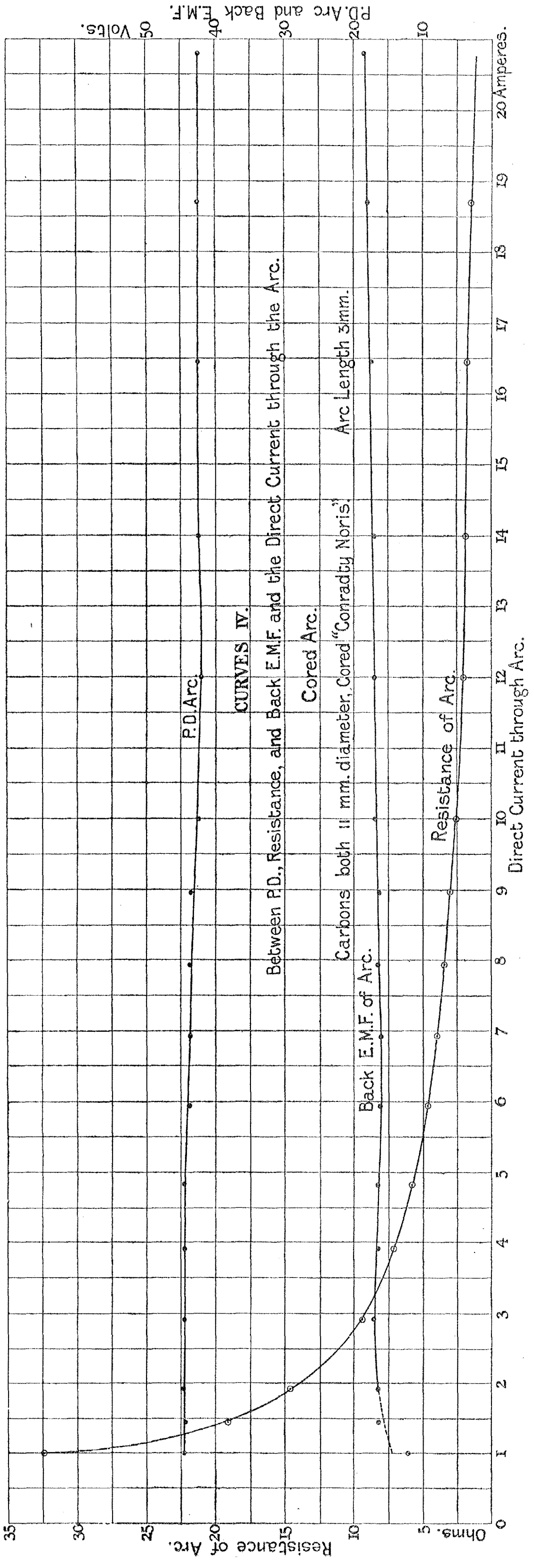
* 'Journal of the Institution of Electrical Engineers,' 1901, vol. 30, p. 248.

of 3 millims. with different values of the direct current. The results of these tests for solid and cored arcs are plotted in Curves III. and IV. The frequency of the



testing current was 100,000 ∽ per second with solid arcs, and 26,000 ∽ with cored arcs. In every experiment the power-factor of the arc was also determined, and found equal to + 1 to within the limits of experimental error, except for a very few small current arcs whose resistance was so high that the power-factor could not be measured with any certainty.

In the Curves III. and IV. there is plotted, besides the resistance of the arc and the P.D. between its terminals, the back E.M.F., calculated as the difference between the P.D. and the product of the direct current into the resistance, or ohmic drop in the arc.

The resistance of both the cored and the solid arcs increases with decrease of the direct current, apparently tending to become infinite for current zero. The back E.M.F. of the solid arc first decreases with increase of current and then slightly increases again, having a minimum of 11·3 volts at about 6 ampères. With cored carbons the back E.M.F. increases with increase of current, from 12·2 volts at 1 ampère to 18·5 volts at 20·8 ampères. It is curious to note that the back E.M.F. of *solid* arcs is larger than that of *cored* arcs for small currents, the reverse being the case with the larger currents.

As the back E.M.F. does not vary much for any of the arcs, the whole of the values observed being between 11·2 volts and 18·5 volts, the high P.D.'s required to maintain very small-current solid arcs is mainly due to the resistance of the arc, and not to the change in its back E.M.F.

The connection between the resistance r and the current A for the *cored* arc, length 3 millims., between 11-millim. "Conradty Noris" carbons, can be approximately expressed over the range from 1·5 ampères to 20 ampères by the very simple relation $(r + 0{\cdot}25)\text{A} = 29$.

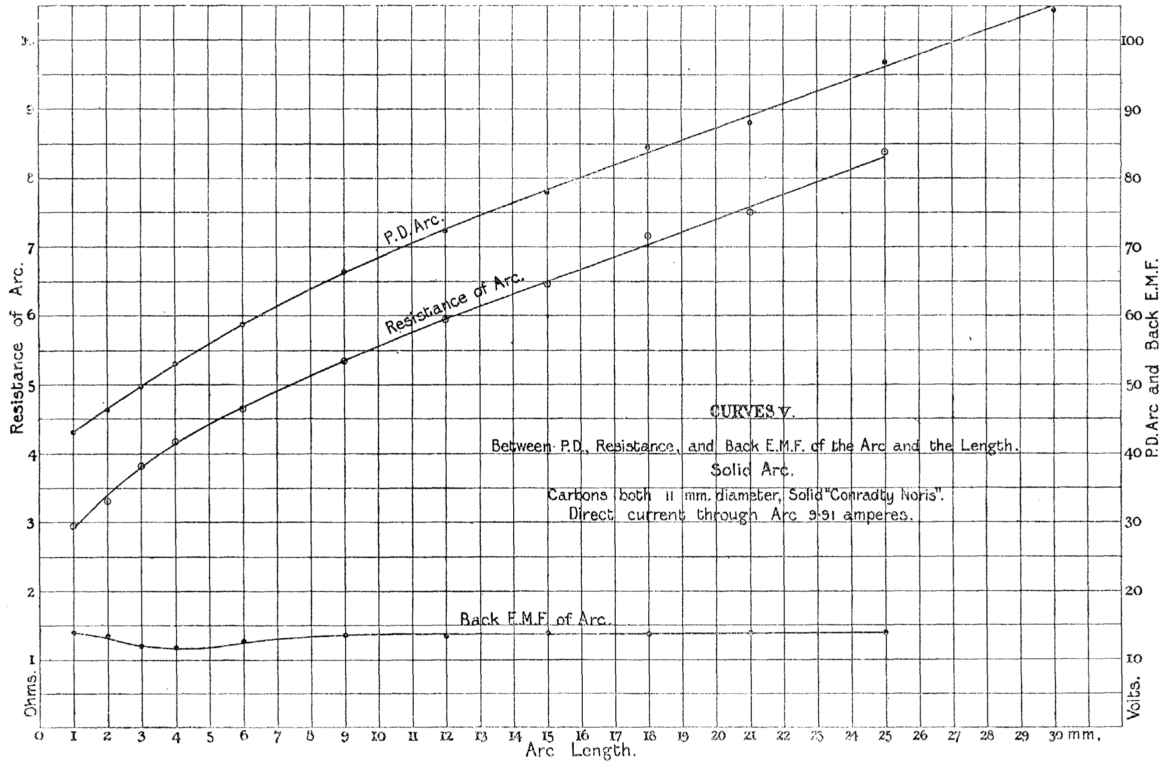
For the *solid* arc, length 3 millims., between 11-millim. "Conradty Noris" carbons, no such simple relation seems to exist; but the curve may be approximately represented over the range 1·5 ampères to 11 ampères by the relation $r = 33{\cdot}5\text{A}^{-1} + 42\text{A}^{-2}$.

Effect of Varying the Arc Length.

The direct current through the arc being kept constant, the connection between the back E.M.F., the resistance, and the length, is given in Curves V. and VI.

With both *solid* and *cored* arcs the effect of increasing the length is to increase the resistance, though not proportionately to the length, the curve between resistance and length being very similar to that between P.D. arc and length. This latter curve is generally assumed to be a straight line, but such is not the case over the wide range of lengths 1 millim. to 30 millims. used in these experiments (see Appendix I.).

The back E.M.F. of the *solid* arc is nearly independent of the length, dropping slightly to a minimum at 4 millims. and then rising again. With the *cored* arc the back E.M.F. decreases with increase of length.



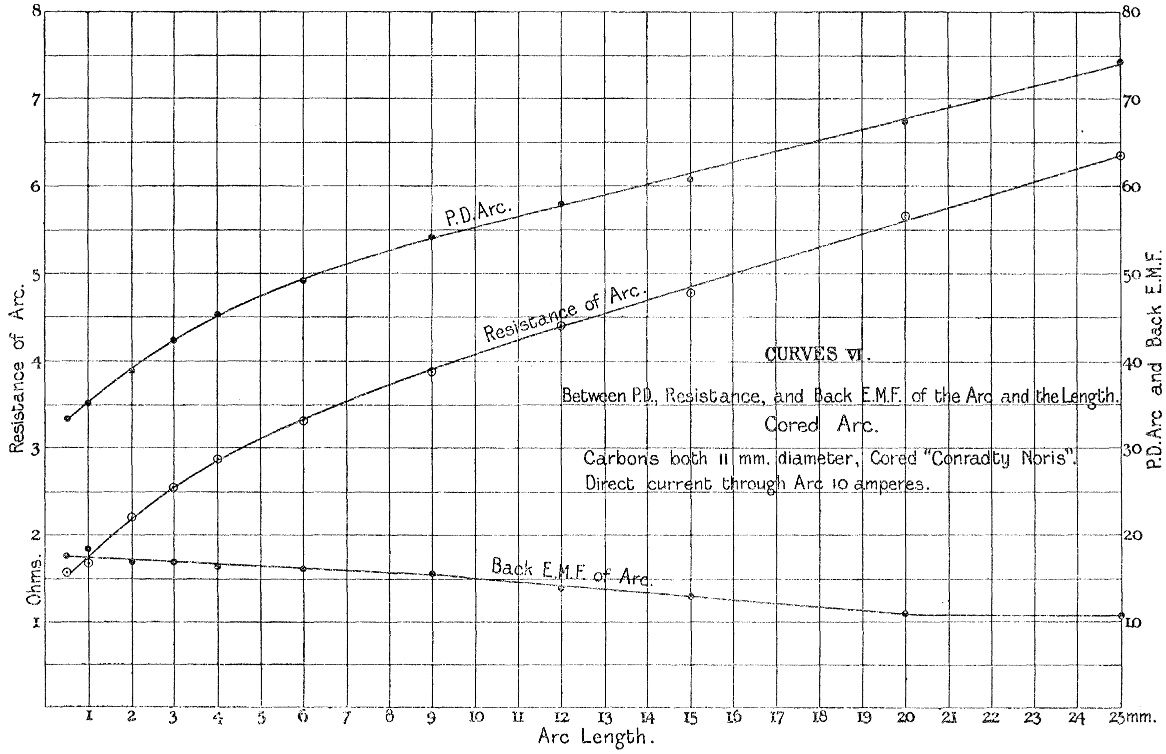
CURVES V.

Between P.D., Resistance, and Back E.M.F. of the Arc and the Length.

Solid Arc.

Carbons both 11 mm. diameter, Solid "Conradty Noris".

Direct current through Arc 9·91 amperes.



CURVES VI.

Between P.D., Resistance, and Back E.M.F. of the Arc and the Length.

Cored Arc.

Carbons both 11 mm. diameter, Cored "Conradty Noris".

Direct current through Arc 10 amperes.

Effect of Varying the Nature of the Electrodes.

The results of experiments to determine whether the resistance and back E.M.F. depend on the size and nature of the electrodes are given in Table IV. The effect of the size of the electrodes is not well marked in the case of "Conradty" and "Apostle" carbons, the changes in size being probably too small to make the effect very evident; the resistances of the arcs are, however, slightly larger with the smaller carbons. With "Le Carbone" Electrographitic solid carbons the impedance of the arc between two 11-millim. carbons is about 17 per cent. higher than that between two 9-millim. carbons. Observations on the arc between these two 11-millim. carbons, which was very unsteady and difficult to maintain, lead the author to think that the observed difference in impedance is not due to the change in size of the electrodes, but to the material of the two sizes of carbons being different, though nominally the same. It is also to be noted that the power-factor, 0·92 of the arc between the 11-millim. carbons, is the only one which has not been found equal to + 1, to within the limits of experimental error, at a frequency of 100,000 ∽ per second. To be quite certain that this was not owing to some error the experiment was repeated, but with practically identical results.

It may be mentioned that these Electrographitic carbons are not ordinary arc lamp carbons, but were specially made for the experiments by "Le Carbone." They are supposed to consist of pure graphite, and they are said to be made by expelling the remaining impurities from carefully prepared carbon by heating it in an electric furnace until the impurities are volatilised.

Both the resistance and the back E.M.F. of the arc depend greatly on the make of carbon, that is to say, on the composition of the electrodes, since it is very improbable that any two makers' carbons have identical chemical composition. The experiment of soaking a pair of solid "Conradty Noris" carbons in potassium carbonate, drying, and re-determining the resistance, shows that the effect of introducing this potassium salt was to reduce the resistance from 3·81 ohms to 2·92 ohms, and to increase the back E.M.F. from 12 volts to 15 volts for the same arc length and current. A similar effect is produced by drilling out one of the carbons and inserting a glass rod as a core, probably due to the introduction of sodium into the vapour column. The lower resistance and higher back E.M.F. of arcs between cored carbons than of those between solid carbons is also probably due to a similar cause, namely, the presence of potassium silicate in the core. In fact, it seems probable that the whole of the observed differences between solid and cored arcs, and between arcs for which different makes of carbons are used, not only in resistance and back E.M.F., but also in all their physical properties, are due to the different amounts of the traces of foreign substances present in the arc.

The author believes that if it were possible to obtain *perfectly pure carbon* electrodes, then the resistance of the arc between them would be very high, so high that it might be impossible to maintain a true arc between them at all. He is of the opinion that traces of impurities, such as the vapours of the alkaline earths, are essential to provide the carriers of the electric charges in the vapour column, so as to render it conducting and the electric arc as we know it a physical possibility. Unfortunately it has not, up to the present, been possible to obtain pure carbon electrodes in order to test this theory. In favour of it is, however, the known fact that, given an arc of fixed length and current between the best commercial solid carbons, then any addition to it of such substances as potassium or sodium reduces the P.D. required to maintain the arc and its resistance and increases its stability.

The difference between the 11-millim. and 9-millim. Electrographitic carbons mentioned above is probably caused by the last traces of impurities having been more completely expelled in the manufacture from the 11-millim. size than from the 9-millim.

TABLE IV.—Various Carbon Electrodes.

Arc length 3 millims. Added alternating currents 0·1 ampère
Direct current through arc 9·91 ampères. Frequency 100,000 ∼ per second.

Nature of Electrodes Varied.

Make and description of carbon electrodes used.	Diameter.		Direct P.D. arc.	Resistance of arc.	Power-factor of arc.	Resistance of arc × current.	Back E.M.F. of arc.	Remarks.
	+	−						
	millims.	millims.	volts.	ohms.		volts.	volts.	
"Conradty Noris" solid .	11	11	49·8	3·81	$0\cdot99_2$	37·8	12·0	
" " " .	11	9	49·8	3·83	$0\cdot99_9$	38·0	11·8	
" " " .	9	9	50·8	3·90	$0\cdot99_1$	38·6	12·2	
"Apostle" solid . . .	11	11	49·3	4·05	1·00	40·1	9·2	
" " . . .	11	9	49·9	4·07	$0\cdot99_7$	40·3	9·6	
"Brush" solid	11	11	50·6	4·04	$0\cdot99_1$	40·0	10·6	
"Le Carbone" solid . .	11	11	50·4	4·26	1·00	42·2	8·2	
"Le Carbone" . . . }	11	11	51·5	4·66	$0\cdot91_7$	46·1*	5·4	Very unsteady arc.
Electrographitic. . . }	11	9	51·2	4·45	$0\cdot99_1$	44·1	7·1	Fairly steady arc.
Solid . . . . }	9	9	50·1	3·95	1·00	39·1	11·0	Very steady arc.
"Conradty Noris" solid, soaked in 10 per cent. solution of K_2CO_3 for 36 hours and dried .	11	11	44·5	2·92	1·0	28·9	15·6	Potassium rapidly burnt out of carbons, results are means of two sets of readings only.
"Conradty Noris" solid, centre of negative drilled out and filled up with a glass rod 2·5 mlliims. diameter	11	11	33·1	2·08	$0\cdot98_5$	20·6	12·5	Very unsteady arc, results are means of 6 sets of readings.
Arc replaced by 3·5 ohm resistance, described page 317	—	—		3·50	$0\cdot99_9$	—	—	

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* This is an impedance, as the power-factor is not unity, the only one not found to be unity within the limits of experimental error.

Seat of the Back E.M.F. Search Carbons in the Arc.

The fact that the arc has a back E.M.F. which appears to increase with the amount of foreign substances present in the vapour column, at once leads to the question whether this E.M.F. is located at one or the other of the electrodes, or distributed along the vapour column. In order to obtain an answer to this question, some experiments were made on a 6-millim. 9·91 ampère *solid* arc by introducing a search carbon, 2 millims. diameter, into the arc, and measuring not only the direct P.D. between the search carbon and each of the main carbons, but also the impedance to the high-frequency testing current of that part of the arc between it and each of the main carbons.

In the experiments, three different positions of the search carbon were employed, (1) with its centre 1 millim. from the positive electrode, (2) central in the arc, (3) with its centre 1 millim. from the negative electrode. The fine point to which the search carbon burns was always kept, so far as possible, just reaching to the axis of the main carbons. The results of these experiments are given in Table V.

The introduction of a search carbon into an arc always greatly disturbs the conditions of the arc, and the present case was no exception. The introduction of the search carbon increased the direct P.D. arc by 4·0 volts, and the impedance of the arc lamp by 0·44 ohm. So that the introduction of the search carbon, either by deflecting the arc and so increasing its length, or by chilling the vapour column, increases its resistance by an amount which approximately accounts for the observed increase in P.D. arc. The back E.M.F. of the arc, as a whole, was but little affected by the introduction of the search carbon. This distortion of the arc by the search carbon probably also accounts for the observation that the measured impedance of the arc as a whole is not equal to the sum of the impedances of the two parts comprised between the search carbon and the main electrodes.

Owing to the correct method of apportioning between the two electrodes, the resistance and self-induction of the loop formed by the carbons, holders, and frame of the lamp, being unknown; and owing to the fact that the measured quantities are only roughly approximate, due to the disturbing effect of the search carbon, no attempt was made to apply the small correction to the observations for the self-induction and resistance of the carbon holders and lamp frame, and the observed impedances were treated as resistances, and the back E.M.F.'s calculated as usual. Further, the three arcs which had the same length and current will be considered as having been identical, though such was not strictly the case.

On these assumptions, consider the resistance between the positive electrode and the search carbon when the search carbon is 1 millim. from the positive electrode, and then 5 millims. from the positive electrode (*i.e.*, 1 millim. from the negative). The change in resistance due to this change of 4 millims. in the position of the electrode is 1·72 ohms. Taking next the measurements made between the negative electrode and the search carbon, the difference for the same movement of the search carbon is 1·64 ohms. The mean of these two results is 1·68 ohms for a movement of the search carbon of 4 millims. If this distance really represented the length of the vapour column between the two positions of the search carbon, and if its resistance is uniform, then its resistance per millim. would be 0·42 ohm. It is probably less than this, owing to the length of the vapour column between the two positions being appreciably longer than 4 millims., due to its distorted shape.

TABLE V.—Search Carbon in Arc.

Carbons both 11 millims. diameter. Solid "Conradty Noris."

Arc length 6 millims. Direct current through arc 9·91 ampères.

Added alternating current 0·1 ampère. Frequency 100,000 ∼ per second.

Position of search carbon.	Direct P.D. arc lamp.	Direct P.D. between search carbon and + electrode.	Direct P.D. between search carbon and − electrode.	Impedance of arc lamp.	Impedance between search carbon and + electrode.	Impedance between search carbon and − electrode.	Impedance of arc lamp × direct current.	Impedance between search carbon and + electrode × direct current.	Impedance between search carbon and − electrode × direct current.	Back E.M.F. of arc lamp.	Back E.M.F. between search carbon and + electrode.	Back E.M.F. between search carbon and − electrode.
	volts.	volts.	volts.	ohms.	ohms.	ohms.	volts.	volts.	volts.	volts.	volts.	volts.
One millim. from + electrode .	64·3	37·8	25·3	$5\cdot 3_1$	$2\cdot 1_4$	$3\cdot 2_1$	$52\cdot{}_6$	$21\cdot{}_2$	$31\cdot{}_8$	$11\cdot{}_7$	$16\cdot{}_6$	$6\cdot{}_5$
Central . . .	62·8	43·9	18·9	$5\cdot 0_5$	$2\cdot 6_2$	$2\cdot 5_5$	$50\cdot{}_0$	$26\cdot{}_0$	$25\cdot{}_3$	$12\cdot{}_8$	$17\cdot{}_9$	$6\cdot{}_4$
One millim. from − electrode .	63·8	53·9	10·3	$5\cdot 1_8$	$3\cdot 8_6$	$1\cdot 5_7$	$51\cdot{}_3$	$38\cdot{}_3$	$15\cdot{}_6$	$12\cdot{}_5$	$15\cdot{}_6$	$5\cdot{}_3$
Mean values .	63·6	—	—	$5\cdot 1_8$	—	—	$51\cdot{}_3$	—	—	$12\cdot{}_3$	$16\cdot{}_7$	$6\cdot{}_1$

Assuming the resistance to average 0·42 ohm per millim., then the resistance of the 6 millims. of *vapour column* is only 2·5 *ohms*, as against 5·2, the measured value for the lamp and arc. There must therefore be a large resistance at or near the electrodes. Calculating its value from each of the three experiments, the resistance at or near the *positive* electrode is 1·72, 1·36 and 1·76 ohms respectively, mean 1·61 *ohms*; and the resistance at or near the *negative* electrode is 1·11, 1·29 and 1·15 ohms respectively, mean 1·18 *ohms*. As there seems no very good reason to suppose that the resistance of the vapour column is very much greater near the electrodes than elsewhere, the above resistances are probably located at the contact between the electrodes and the vapour column.

The resistance of the arc as a whole may therefore be considered to consist of three parts, the resistance of the vapour column and the resistances of the two contacts between it and each of the electrodes.

The mean value of the back E.M.F. of the arc lamp, with the search carbon in place in the arc, *i.e.*, 12·3 volts, is practically identical with its value of 12·7 volts, obtained without any search carbon in the arc. The back E.M.F. of that part of the arc between the search carbon and the positive electrode had, for the three different positions of the search carbons, the values 16·6, 17·9 and 15·6 volts respectively, which may be considered as indicating that this back E.M.F. is independent of the position of the search carbon to within the limits of accuracy of the present experiments. The back E.M.F. between the search carbon and the negative electrode calculated in the same way, *i.e.*, by subtracting the product of resistance by direct current, or ohmic drop in this part of the arc, from the observed direct P.D. between the search carbon and the negative electrode, is a *negative quantity*, so that there exists a *forward* E.M.F. which helps the flow of the direct current. The value of this forward E.M.F. was 6·5, 6·4 and 5·3 volts respectively, again practically independent of the position of the search carbon.

These experiments show that there is in this arc at or near the positive electrode a *back* E.M.F. having a mean value of 16·7 volts, and at or near the negative electrode a *forward* E.M.F. of 6·1 volts. The fact that the algebraic sum of these two voltages is not equal to the observed mean back E.M.F. 12·3 volts of the arc, as a whole, is probably caused by the distortion of the lines of flow of the current through the arc, produced by the introduction of the search carbon, which renders the sum of the resistances measured between each of the main electrodes and the search carbon greater than their total as measured between the main electrodes.

The back E.M.F. of the arc as a whole consists, therefore, of two E.M.F.'s located at or near the electrodes, the larger E.M.F. situated at or near the positive electrode opposing the flow of the current, the smaller E.M.F. situated at or near the negative electrode helping the flow of the current.

The approximate values of the resistance E.M.F. of each part of the arc, and the drop of volts due to each of these causes, corrections for the self-induction and resistance of the holders and carbons being omitted as before, are given in Table VI. The P.D.'s in this table multiplied by the direct current of 9·91 ampères give the power supplied to each part of the arc.

TABLE VI.—Distribution of the P.D. in a Solid Arc.

Carbons, both 11-millims. "Conradty Noris." Arc length 6 millims. Direct current through Arc 9·91 ampères.

	Ohmic drop due to resistance.	Back E.M.F.	Sum of back E.M.F. and ohmic drop.
	volts.	volts.	volts.
At or near the + crater	+16·0	+16·7	+32·7
Vapour column	+25·0	0	+25·0
At or near the crater	+11·7	− 6·1	+ 5·6
Total	+52·7	+10·6	+63·3

Conclusion.

Ever since the arc was first assumed to have a back E.M.F., speculation has been rife as to its cause and its location in the arc. So long as the value assigned to it was of the order of 40 volts, and it was supposed to be located only at the crater surface, there was great difficulty in offering any consistent explanation of it. It remains, therefore, to consider whether the new facts set forth in this paper render the matter more susceptible of a satisfactory explanation.

So far as the resistance of the arc is concerned, there seem to be no difficulties, the known relations between the size and shape of the vapour column, the size of the craters, the current, and the arc length, explain the observed changes in resistance when the two latter variables are altered. The magnitude of the resistance of the vapour column and of the contacts between it and the electrodes are not such as to offer serious difficulties, nor does the fact that they are altered by the presence of foreign bodies.

Any explanation of the back E.M.F. of the arc as a whole must, in the light of the results given in the last section, account for the existence of two unequal E.M.F.'s of opposite signs, of which the larger opposes the flow of the current. The observed back E.M.F. of the arc is the resultant of these two, their values being of the order of 17 volts and 6 volts respectively in the solid arc. The existence of two E.M.F.'s of opposite signs, situated at or near the electrodes, considerably simplifies matters, since any explanation which would account for a back E.M.F. in the direction carbon-to-vapour would probably also explain a forward E.M.F. vapour-to-carbon. These E.M.F.'s are probably either due to a polarisation at the electrodes or to thermo-electric forces. The polarisation E.M.F.'s include those due to chemical changes and those which have been assumed to be caused either by the volatilisation or the tearing-off of particles from the electrodes. There appears to be very little evidence in favour of these last two explanations.

The statement sometimes made that, as it requires a certain amount of work to be done to convert the solid electrodes into the gaseous state, and as this work is done by the current, therefore the current must be flowing against a back E.M.F., is without sufficient foundation. It is true that, in the above case, a P.D. will exist which opposes the flow of the current, but that this P.D. is a reversible phenomenon, and therefore an E.M.F., is not necessarily the case. The volatilization of the electrode is of course reversible, but it requires experimental evidence to prove that it is accompanied by a corresponding reversible electric phenomenon, so that the energy, supposed supplied electrically, to cause the volatilisation, tends to be returned, on condensation, in the form of electric energy. There is no evidence that any appreciable E.M.F. is produced by the tearing-off of the solid particles from the electrodes, and HERZFELD's experiment of attracting these particles out of the arc by means of an electrostatic field, which he says did not affect the P.D. or current through the arc, seems to indicate that the back E.M.F. of the arc cannot be due to this cause.

That more than a small part of the back E.M.F. is due to a polarisation, such as occurs in an ordinary cell, is difficult to conceive. If such were the case, what is the nature of the chemical compound produced, and what becomes of it? It is certain that practically the whole of the energy supplied to the arc is emitted again as light and heat, and that there is no considerable portion stored up in the products produced by the arc. If, therefore, any chemical change takes place at the positive electrode accompanied by absorption of energy, this energy must be given out again in the arc or flame, and the reverse chemical change take place. Some of the energy might be given out at the negative electrode, and account for the forward E.M.F. observed there. The nature of the substance in which the chemical change must take place, which change reverses and so forms a cyclic process, is an almost insuperable difficulty, since the rate at which the energy must be constantly absorbed and given out again is considerable, and the materials present in which this chemical change must take place are only carbon, its vapour, and the slight trace of impurities which the author thinks essential to the existence of the arc.

If the impurity be assumed to be a salt of potassium, there is the possibility that a carbide of potassium might be formed, and that part of the P.D. might be due to the arc forming a cell, having carbon and potassium carbide as its electrodes, and the vapour column as the electrolyte, the products produced by the flow of the current through the cell being conceived to be destroyed in the flame which surrounds the arc proper.

That a chemical combination between the carbon and surrounding gas is not the cause of the back E.M.F., at any rate in the normal silent arcs considered here, is evident from the fact that the arc is but little affected by the nature of the gas in which it burns, as has been shown by Professor S. P. THOMPSON and others. Putting aside the possible combination of the carbon with the slight trace of impurity, which may account for a small part of the back E.M.F., the polarisation E.M.F.'s do not seem able to account satisfactorily for the whole back E.M.F. of the arc, without making assumptions which are at present unsupported by any satisfactory evidence.

Against the back E.M.F. of the arc being due to thermo-electric forces, it is generally urged that these forces are usually reckoned in tenths and hundredths of a volt, and that therefore the order of magnitude of the back E.M.F., which was then supposed to be about 40 volts, rendered it highly improbable that it was due to thermal causes.

There is no doubt that the temperature of the positive crater is very high—said to be about 3500° C., and it is quite possible that there may exist large temperature gradients near the electrodes, since the true back E.M.F. to be accounted for is only of the order of 17 volts at the positive crater. It is therefore worth while re-considering whether after all the back E.M.F. of the arc may not be due to thermo-electric forces.

In this connection the experiment of DUBS* is of considerable interest. He took two carbon plates, about 1 millim. apart, and caused a blow-pipe flame to impinge across their edges, and found that a small current could be obtained from one plate to the other through a galvanometer. He considered this result might be analogous to the back E.M.F. of the arc. A similar experiment has also been described by OLIVETTI.†

The views of the author that impurities, such as the salts of the metals of the alkaline earths, are essential to the existence of the arc, have led him to try a considerable number of experiments on the P.D.'s produced by unequally heating carbon electrodes, either with the addition of such salts to the electrodes, or to the flame used as a source of heat. Most of these experiments were carried out by heating the tips of ordinary arc-lamp carbons held in the hand-feed arc lamp already described, by causing a "blow-through" or a "mixed" oxy-coal-gas flame to impinge upon them.

The terminals of the arc lamp were connected to a direct-current voltmeter and were not joined in any way to a source of current, so that the P.D.'s observed were not due to leakage from any extraneous source. The choice of the voltmeter presents some difficulty. At first sight, owing to the high resistance of the flame between the two tips of the carbons, it might seem that an electrostatic or a very high resistance voltmeter would be the best. Electrostatic voltmeters were however found unreliable, owing to the friction of the gases in the jet and against the carbons charging them up electrostatically. Two voltmeters were therefore used, the one an ordinary Weston pivot instrument, having a resistance of about 600 ohms, and the second

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* 'Beiblätter,' 1889, vol. 13, p. 197.

† 'Electrical Review,' 1892, vol. 31, p. 728.

a reflecting moving-coil instrument of 20 ohms, having a resistance of about 16,000 ohms in series with it. The resistance of both these instruments is so low that no deflection due to the frictional charges could be observed on them. By comparing the P.D.'s obtained when one or the other of the voltmeters was used an estimate of the resistance of the vapour between the carbon tips could be made.

If the flame was caused to impinge on the two carbon tips so as to heat them equally, as judged by eye, then no P.D. was observed between them. If the flame was now moved so as to heat one carbon more than the other, then a P.D. was observed between the carbons, the hotter being positive to the cooler as indicated by the voltmeter, that is, in the same direction as the back E.M.F. of the arc, assuming that the positive crater is the hotter.

The highest P.D. obtained was 1·5 volts when using cored carbons which had been previously soaked in potassium carbonate solution. This P.D. was not in any way due to differences in the quality of the two carbons, since by moving the jet so as to heat either carbon more than the other the P.D. changed sign, the hotter carbon always being positive. By setting one carbon in front of the other it was found that the direction of the P.D. was unaffected by whether the stream of flame gases flowed from the hotter to the cooler carbon or *vice versâ*.

With two pieces of the same solid carbon, about 2 millims. apart, no foreign bodies being introduced in any way, the highest P.D. obtained was about 0·5 volt on the higher resistance voltmeter, and the resistance of the flame between the carbons was deduced as about 4000 ohms, so that the E.M.F. between these *solid* carbons was only about 0·62 volt. After soaking this same pair of carbons in potassium carbonate the P.D. obtained was 1·0 volt., the resistance of the vapour being only a few hundred ohms, so that the effect of introducing this potassium salt was to greatly increase the E.M.F. between the carbons. If these E.M.F.'s are due to Peltier effects, then it would seem as if the introduction of the potassium greatly increased the thermo-electric power of the junction.

The above experiment is in agreement with the fact that the back E.M.F. of the cored arc has been generally found larger than that of the solid arc, as the cored arc undoubtedly contains more impurities.

By varying the proportions in which the oxygen and coal gas were mixed before burning, it was found that the highest P.D.'s appeared to correspond with the gases being burnt in their combining proportions, so as to produce the hottest flame; if either gas was considerably in excess, a much lower P.D. was obtained. It is to be noted, however, that by suppressing the coal gas altogether and allowing the carbon to burn in the oxygen, P.D.'s up to 1·5 volt could be obtained, as before the hotter electrode always being positive to the voltmeter. These latter experiments, though not conclusive, are not in favour of the P.D. being due directly to a combination of the carbon and either of the gases.

The temperature of the positive crater of the arc is much higher than the highest temperature obtained by the oxy-coal-gas flame, and the temperature differences and gradients which may exist in the arc are probably many times greater than those obtained in any of the above experiments, for it must be remembered that the carbon at the lower temperature must always be at a bright red heat, or else it does not appear to make electrical contact with the flame. It does not therefore seem improbable that the P.D.'s of 1 volt or 1·5 volts obtained by unequally heating the two carbons may have the same origin as the 10-volts to 18-volts back E.M.F. found in the arc, especially as they agree both in direction and in the effect of impurities on them. On this assumption, the probable causes of the back E.M.F. of the arc reduce themselves to two, viz. :—

(1.) A thermo-electric force at the junction of the carbon and vapour, causing the major part of the observed back E.M.F.

(2.) A combination of carbon with the impurity present.

Whether the thermo-electric force at the junction of carbon and vapour be due either to the Peltier effect, or to a high temperature gradient at the contact, *i.e.*, a Thomson effect, or both, it seems at the present time to afford the most satisfactory explanation of the back E.M.F. of the arc and the P.D.'s observed when two carbon electrodes are unequally heated. The main objection is the great difference of thermo-electric power the components of the junction must have, which, if the difference of temperature be assumed to be only 1000° C., amounts to about 15 to 20 times that between bismuth and selenium.

In favour of the view that the back E.M.F. is mainly due to thermal causes may be mentioned the unilateral behaviour of the alternating arcs between a metal ball and a metal point,* between carbons and metals,† and the observation of CROSS and SHEPARD‡ on the effect of cooling the positive crater of the direct-current arc. All these experiments indicate that there is some cause existing which enables the current to flow much more easily up the temperature gradient than down it.

A tentative explanation of the chief causes which oppose the flow of the current through the arc may be given under four heads :—

(1.) The resistance of the true vapour column, which is probably an electrolytic conductor, whose conductivity greatly depends on the traces of impurities, such as potassium and soda, present. Pure carbon vapour, like pure water, has probably a very high specific resistance.

(2.) A high contact resistance between the electrodes and the vapour column, which leads to a large generation of heat, and consequently high temperatures and temperature gradients at these points.

(3.) Possibly a small back E.M.F., due to the electrolytic cell formed by the electrodes and the vapour column as electrolyte.

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* ARCHBOLD and TEEPLE, 'American Journal of Science,' 1891.

† DUDDELL and MARCHANT, 'Journal of the Institution of Electrical Engineers,' 1899, vol. 28, p. 74.

‡ 'American Academy of Sciences,' 1896, p. 227.

(4.) Considerable thermo-electric forces at the contacts between the electrodes and the vapour column, whose resultant is the greater part of the observed back E.M.F. of the arc. The force at the positive opposing the flow of the current, and that at negative helping it, account for the much greater conversion of electric energy into heat at the positive electrode.

In the above, it will be noticed that the volatilisation or sublimation of carbon is not supposed to have any influence on the back E.M.F. except in so far as it limits the temperature the electrodes can obtain.

The introduction of impurities up to a certain limit into the arc will cause the conductivity of the vapour column to increase, and for a fixed current its temperature will probably decrease, so the thermo-forces and the back E.M.F. may be expected to increase as has been observed (see Table IV.).

There is another possible explanation of the larger back E.M.F. when impurities are present, viz., the thermo-electric force of the junction carbon-vapour may be larger the greater the quantity of impurity present. If this is the case, may not the explanation of the drop in P.D., when the arc hisses, or when oxygen or hydrogen come in contact with the positive crater, as found by Mrs. AYRTON, be that *the gas combines with the impurity* and reduces the back E.M.F., and not that it combines with the carbon as suggested by Mrs. AYRTON ?

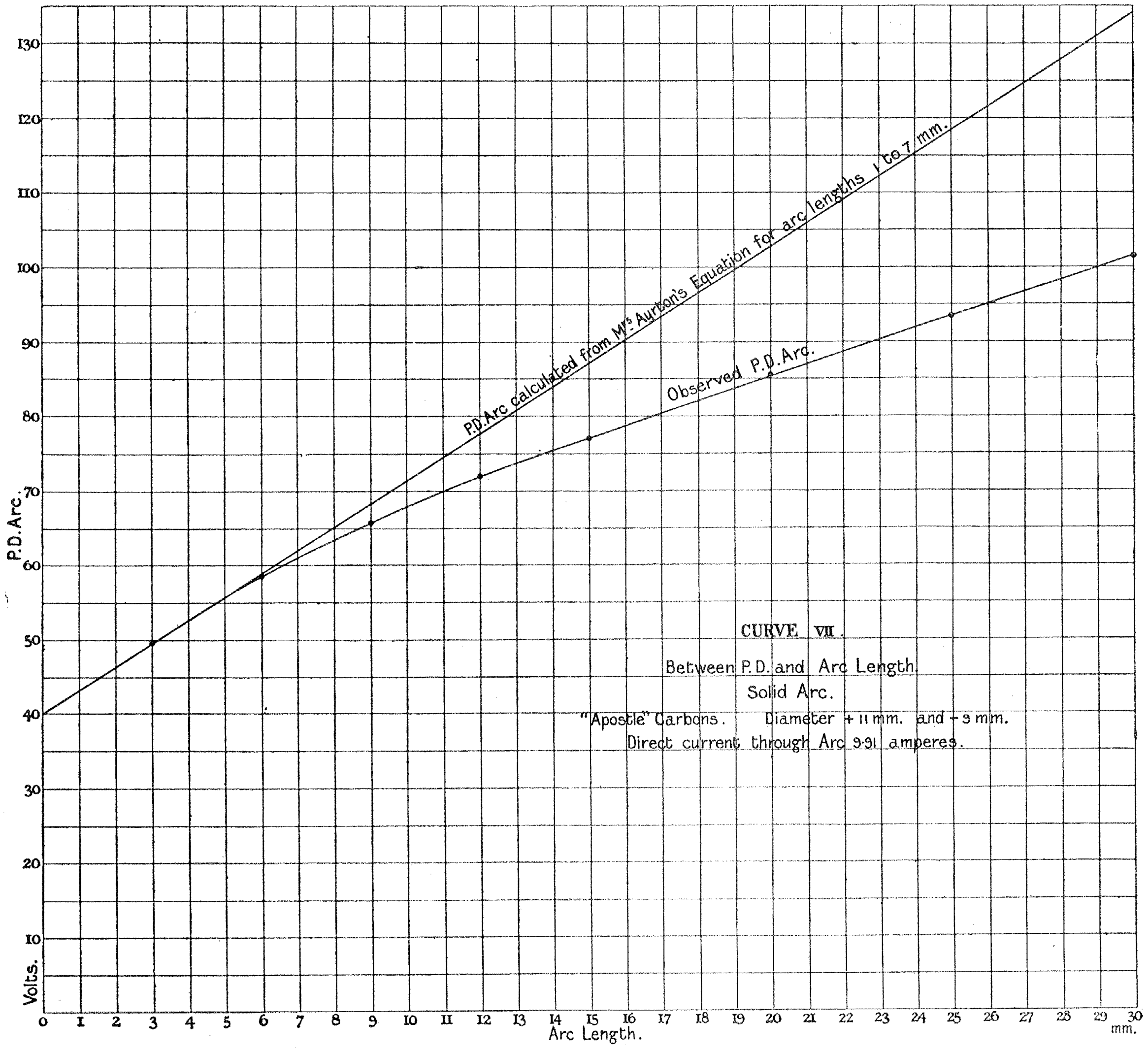
In conclusion, the author wishes to express his thanks to Professor AYRTON and Mr. MATHER of the Central Technical College for the very valuable assistance and advice they have given him during the course of these investigations; he also wishes to thank the many students who have from time to time helped him with the experiments, and especially Messrs. DEL MARR, LYNN, BROWN, WATSON, and VINES.

APPENDIX I.

On the Relation between the P.D. Current and Length of the Arc.

Mrs. AYRTON has pointed out that the relation between the P.D., the current, and the arc length, may for the solid carbons which she used be accurately expressed by an equation of the form $V = \alpha + \beta l + (\gamma + \delta l)/A$, where V and A are the P.D. and current respectively, and α, β, γ, and δ are constants, between the limits over which she varied the current and length. It was thought, therefore, that it would be of interest to determine the values of the constants α, β, γ, δ for the solid "Conradty Noris" carbons used in so many of the experiments in this paper. The two series of results in Curves III. and V. give P.D. when the length is kept constant and the current varied; and also the P.D. when the current is kept constant and the length varied; and in order to increase the accuracy of the equation some extra results were obtained in which both current and length have been varied.

On attempting to find the constants α, β, γ, δ to fit all these values, the first difficulty encountered was that, assuming the current to be constant and the length varied, Mrs. AYRTON's equation required the P.D. to be a linear function of the length; a glance at Curve V. shows that this was *not* even approximately true for these carbons and the long range of lengths from 1 millim. to 30 millims. used. At first it was thought that this might in some way be due to the kind of carbons used in the experiments. The connection between P.D. and length for constant current was, therefore, determined for the same size and make of carbons used by Mrs. AYRTON in her experiments*; the results are given in Curve VII.



In the curve the straight line represented by Mrs. AYRTON's equation is also given. The divergence between the P.D.'s calculated from Mrs. AYRTON's equation and the

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* The 'Electrician,' 1895, vol. 35, p. 420.

observed P.D.'s is very marked, the error in the calculated result being no less than 32·6 volts for an arc length of 30 millims. At the same time this is not due to any difference in the nature of the carbons, since the actual experimental values obtained by Mrs. AYRTON,* and on which her equation is based, fit the Curve VII. with considerable accuracy. The whole of this large difference between Mrs. AYRTON'S equation and the experimental results is due to the fact that the equation is based on the range of arc lengths of from 1 millim. to 7 millims., and that the equation no longer represents the facts if we extrapolate any considerable amount. Even over the range of from 1 millim. to 7 millims. the connection between the P.D. and length for constant current is not accurately a straight line, as can be seen either in Curve VII. or by a careful examination of Mrs. AYRTON'S experimental results.

If the arc length was kept constant and the current varied, a similar difficulty was found with very small currents of 1·5 ampères and lower, namely, that the observed P.D. was less than the calculated P.D., so that Mrs. AYRTON'S *equation can only be considered as approximately representing the facts within the limits she used, and must not be applied to very small currents or long arc lengths.*

Taking, in the present case, as limits from 1·5 ampères to 12 ampères, and from 1 millim. to 6 millims., then the results obtained with the arc between 11 millims. solid "Conradty Noris" carbons can with a very fair approximation be represented by the equation

$$V = 39{\cdot}6 + 1{\cdot}7l + (15{\cdot}5 + 11{\cdot}5l)/A,$$

which is in very close agreement with the equation given by Mrs. AYRTON for solid "Apostle" carbons, 11 millims. and 9 millims. diameter, for the range 1 millim. to 7 millims., viz.:

$$V = 38{\cdot}88 + 2{\cdot}07l + (11{\cdot}66 + 10{\cdot}54l)/A.$$

APPENDIX II.

On the Resistance of an Electrolyte.

In measuring the resistance of an electrolyte by the ordinary Kohlrausch method, using alternating or induced currents, it is usually assumed that the influence of polarisation of the electrode is avoided, and that the frequency of the alternating current used is unimportant, provided that it is moderately high, say a few hundred periods per second. If an appreciable polarisation of the electrodes is produced by the testing current very soon after its application in any given direction, then the

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* 'Electrician,' vol. 35, p. 635.

above assumptions will not be true, and the resistance as measured at such low frequencies will differ from that measured at much higher frequencies.

In order to investigate this matter, the arc was replaced by an electrolytic cell, and its impedance and power-factor tested in a similar way to those of the arc, except that no direct current was sent through the cell. To make the polarisation effects important compared with the ohmic drop, the resistance of the cell must be small, therefore sulphuric acid was chosen as the electrolyte. The cell used was a glass trough 100 millims. long, 104 millims. deep, and having a mean thickness of 15·53 millims. The electrodes were two platinum plates 90 millims. apart (not platinised) which fitted the trough tightly and extended to the bottom.

The mean depth of the electrolyte was 61·6 millims. So that the liquid whose resistance was measured was a rectangular parallelopiped, having a length of 9 centims. and a mean cross-section of 9·57 sq. centims.

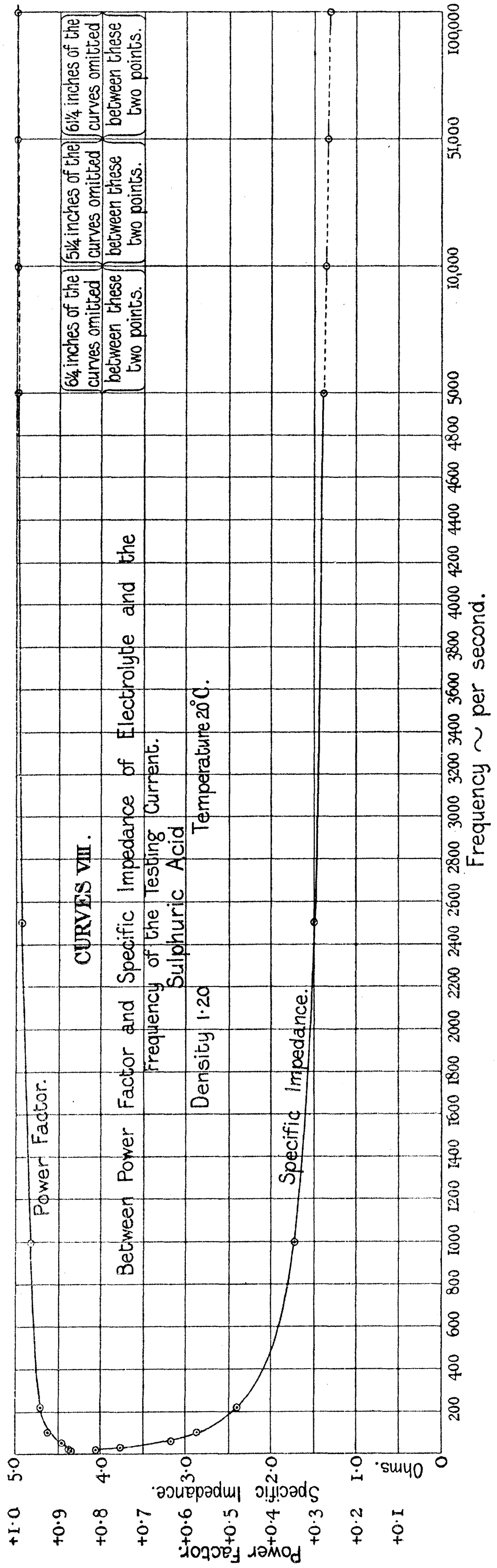
The temperature was kept constant at 20° C. to within 0·1° C. during the test, by immersing the trough in a water-bath. The results of the tests of the impedance and power-factor at different frequencies are given in Curve VIII.

With the above cell, neither the impedance nor the power-factor are independent of the frequency at ordinary frequencies, and it is evident that the polarisation in this electrolyte occurs so rapidly that it is not until the frequency is well above 10,000 ∽ per second that it is unable to follow the variations of the testing current, and that the electrolyte behaves like an ordinary resistance. If the resistance of this cell were tested in the ordinary way, at a frequency of about 100 ∽ per second, the value obtained would be over twice the true resistance of the cell.

The slight drop in impedance observed when the frequency is increased from 50,000 to 100,000 ∽ per second is possibly due to the electrostatic capacity of the plates and liquid in the cell, and to the water-bath surrounding it acting as a shunt to the cell. Attempts were made to determine experimentally the exact value of this correction, which indicated that it was less than 1 per cent. at 100,000 ∽ per second.

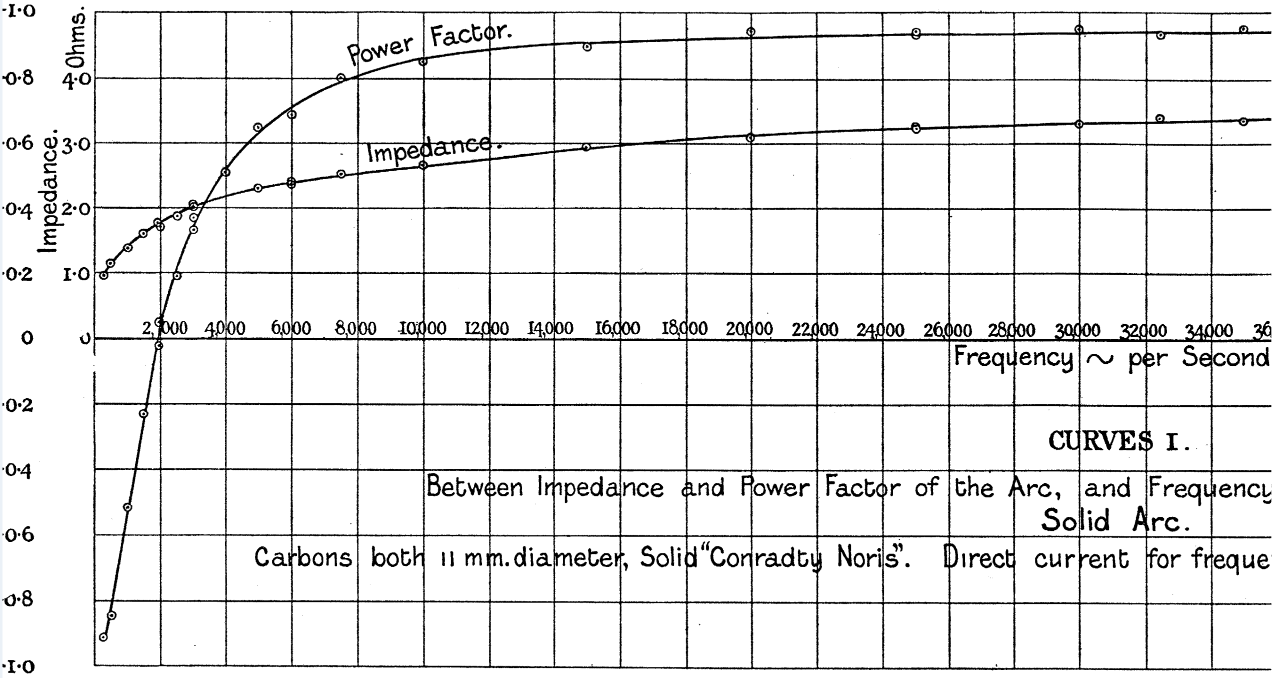
The possible errors due to polarisation, even when using alternating currents, were thoroughly appreciated by KOHLRAUSCH, and in the ‘Leitvermögen der Elektrolyte,’ by KOHLRAUSCH and HOLBORN, means are described for minimising these errors. By interpolation from their results the conductivity of sulphuric acid of the same density and at the same temperature as that used in this experiment is 0·758 (ohm × centim.)$^{-1}$, or a specific resistance of 1·319 × 10^{9} c.g.s., which confirms the value of 1·30 × 10^{9} obtained in the present experiment at a frequency of 50,000 ∽ per second.

The fact that the power-factor is not unity at low frequencies proves that at these frequencies the electrolyte does not behave like an ordinary resistance, and consequently *all* the instantaneous values of the P.D. between the terminals of the telephone sometimes used in the Kohlrausch method cannot be made to vanish at


CURVES VIII.
Between Power Factor and Specific Impedance of the Electrolyte and the Frequency of the Testing Current.
Sulphuric Acid
Density 1·20
Temperature 20°C.
Power Factor.
Specific Impedance.
6¼ inches of the curves omitted between these two points.
5¼ inches of the curves omitted between these two points.
6¼ inches of the curves omitted between these two points.
Frequency ∼ per second.
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
2600
2800
3000
3200
3400
3600
3800
4000
4200
4400
4600
4800
5000
10,000
51,000
100,000
Specific Impedance.
Ohms.
5·0
4·0
3·0
2·0
1·0
0
Power Factor.
+1·0
+0·9
+0·8
+0·7
+0·6
+0·5
+0·4
+0·3
+0·2
+0·1

these frequencies, even supposing the other arms of the bridge quite free from self-induction and capacity. This explains the fact that only a minimum in the sound is obtainable, and not absolute silence.

The conclusion to be drawn from this experiment is that in any case where the P.D. due to the polarisation of the electrodes cannot be made very small compared with the ohmic drop along the liquid whose resistance is being measured, and where the errors due to the polarisation cannot be eliminated by taking two or more tests, then it *must not be assumed without proof that the use of alternating currents at ordinary frequencies of a few hundred periods per second eliminates the possibility of errors due to polarisation.* For in the case of sulphuric acid used above, the polarisation can vary as rapidly as the resistance of the cored arc.

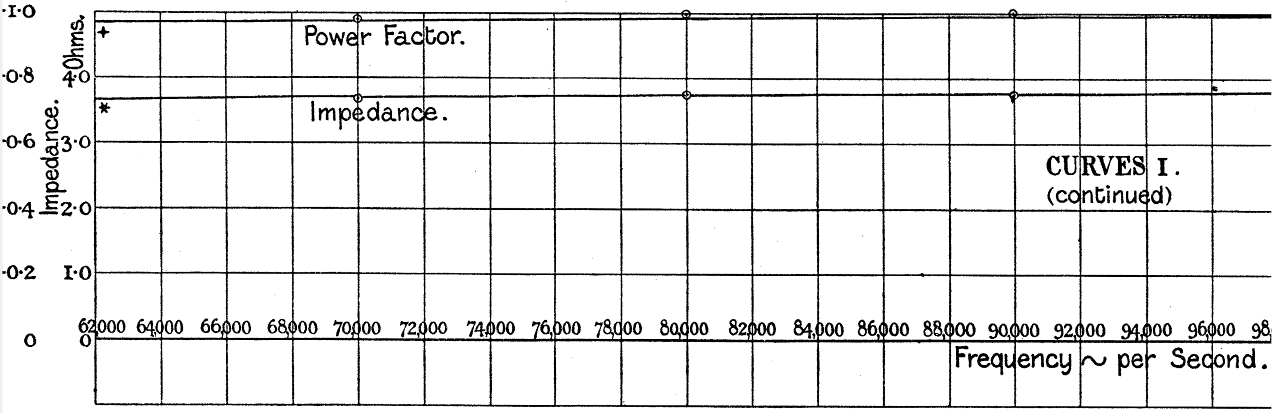


CURVES I.

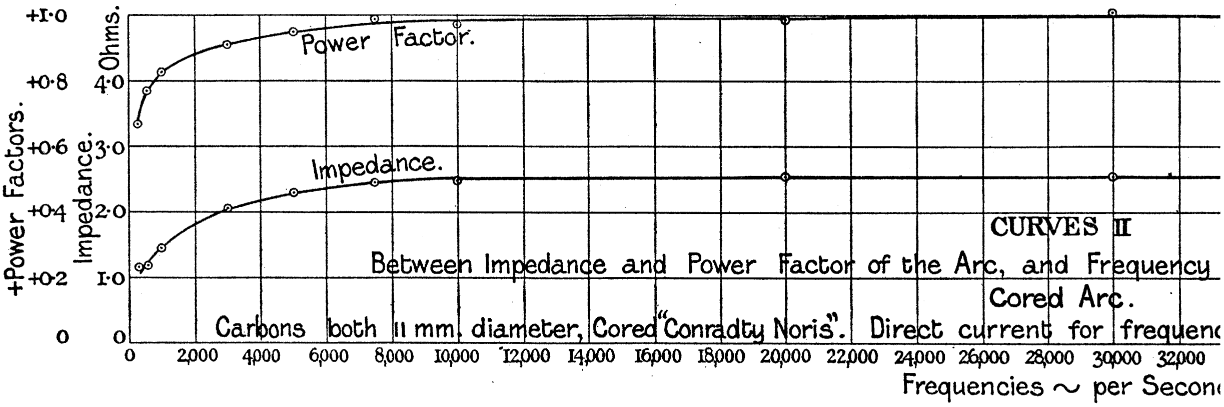
Between Impedance and Power Factor of the Arc, and Frequency

Solid Arc.

Carbons both 11 mm. diameter, Solid "Conradty Noris". Direct current for freque



CURVES I.
(continued)



CURVES II

Between Impedance and Power Factor of the Arc, and Frequency

Cored Arc.

Carbons both 11 mm. diameter, Cored "Conradty Noris". Direct current for frequenc

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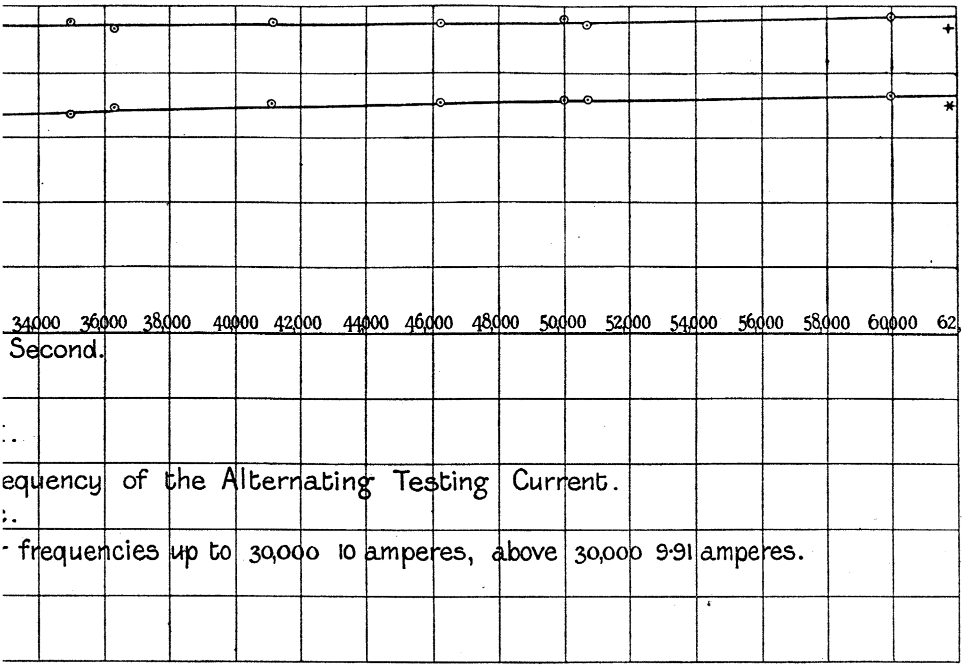
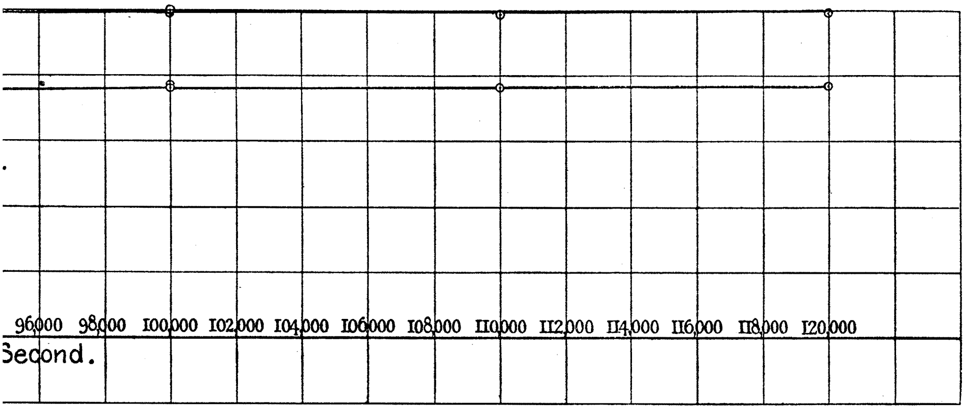
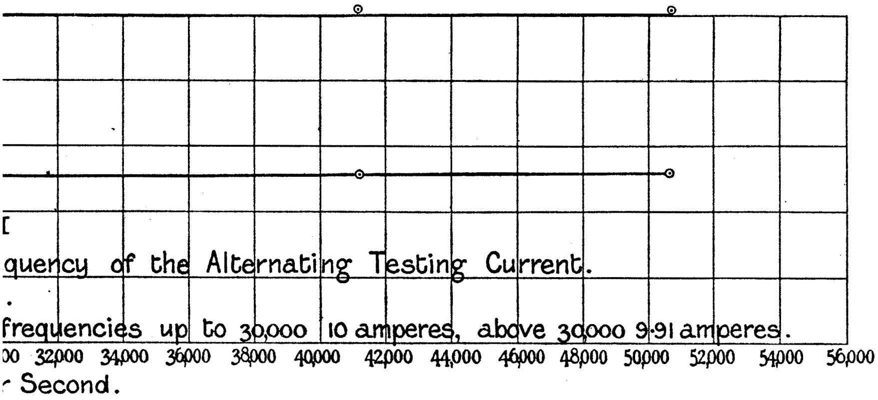
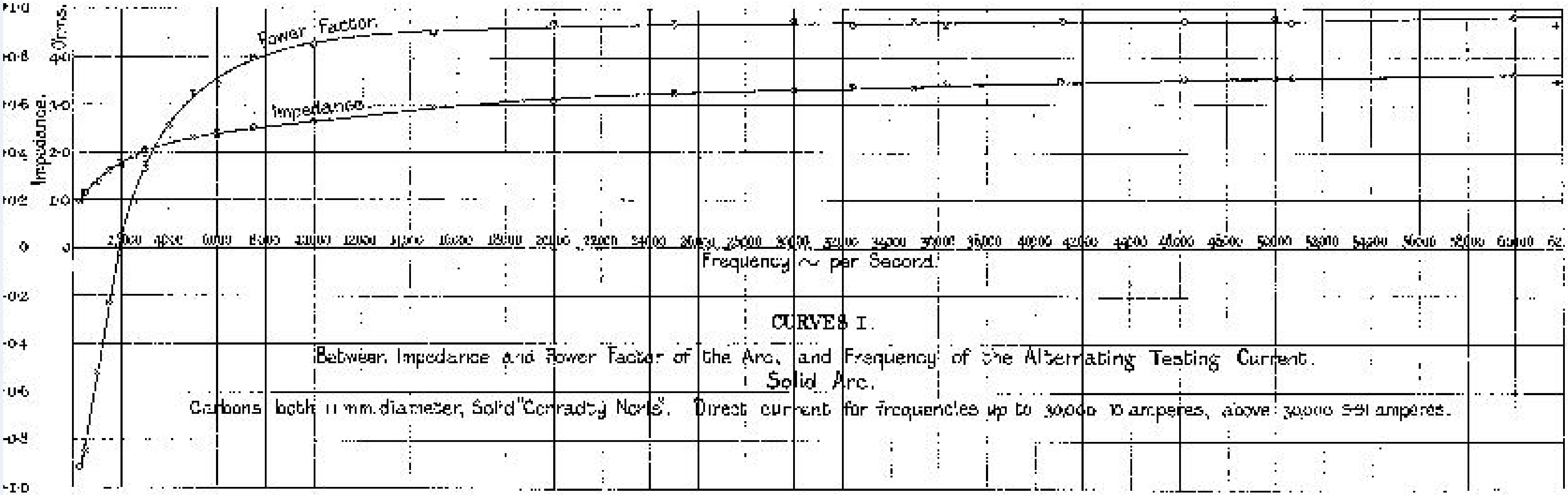






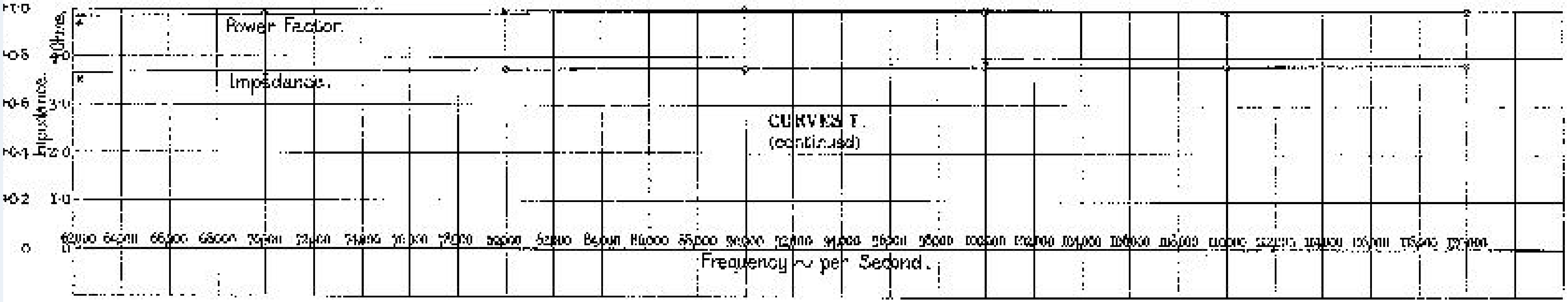


CURVES I.

Between Impedance and Power Factor of the Arc, and Frequency of the Alternating Testing Current.

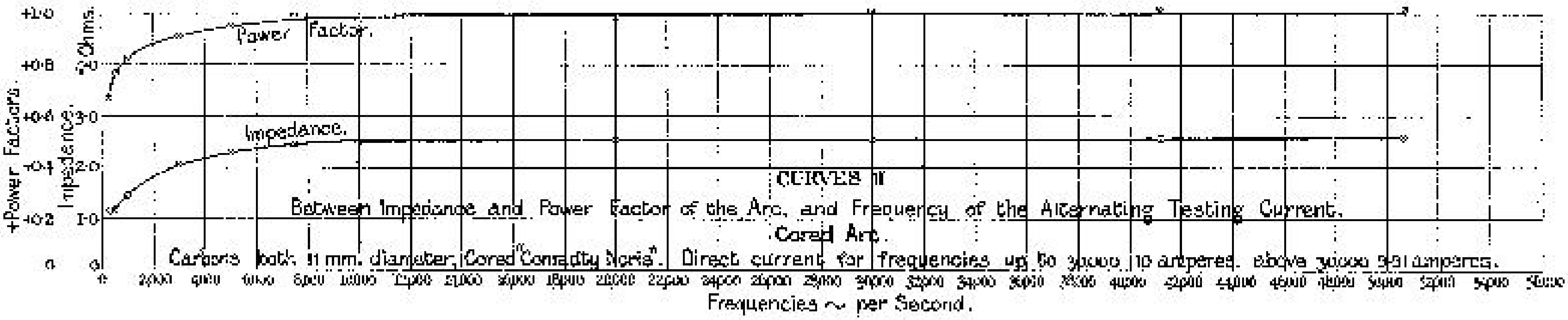
Solid Arc.

Carbons both 11 mm. diameter, Solid "Conradty Noris". Direct current for frequencies up to 30,000 10 amperes, above 30,000 5·91 amperes.



CURVES I.

(continued)



CURVES II.

Between Impedance and Power Factor of the Arc, and Frequency of the Alternating Testing Current.

Cored Arc.

Carbons both 11 mm. diameter, Cored "Conradty Noris". Direct current for frequencies up to 30000 10 amperes, above 30000 5·91 amperes.